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Superhydrophobicity, Spreading and Imbibition

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Biomimetic FSFI'09, Nottingham, UK

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Overview

1. Water Repellency: Concepts

- Insects and surface tension
- Naturally occurring surfaces

2. Topography & Wetting: Theory

- Surface free energy derivations: Wenzel/Cassie-Baxter equations
- Defects and symmetric/random patterns, complex topography

3. Superhydrophobicity: Consequences

- Amplification, attenuation and saturation
- Skating-to-penetrating transition and droplet collapse
- Path definition and droplet motion
- Unexpected superhydrophobicity

4. Porosity, Spreading and Imbibition

- Superspreading, superwetting, hemi-wicking and porosity
- Imbibition and water-repellent soil

Water Repellency: Concepts

Insects at the Water's Surface

Walking on Water



Microcosmos (Copyright: Allied Films, 1996)

Winners and Losers: Understanding provides a competitive advantage

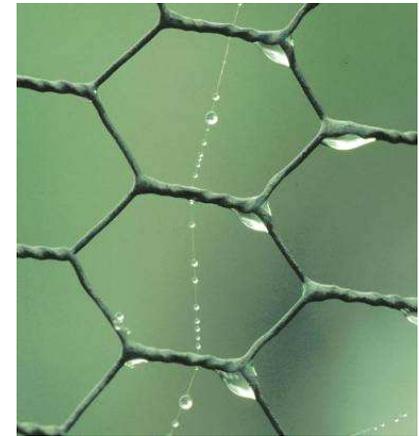
Surface Tension

Liquid Surface

Molecules at a surface have fewer neighbours

Liquid surface ("skin") behaves as if it is in a state of tension

For a free "blob", the smallest area is obtained with a sphere



<http://www.brantacan.co.uk>

Size Matters

Surface tension force \propto length

e.g. Force $\sim R\gamma_{LV}$

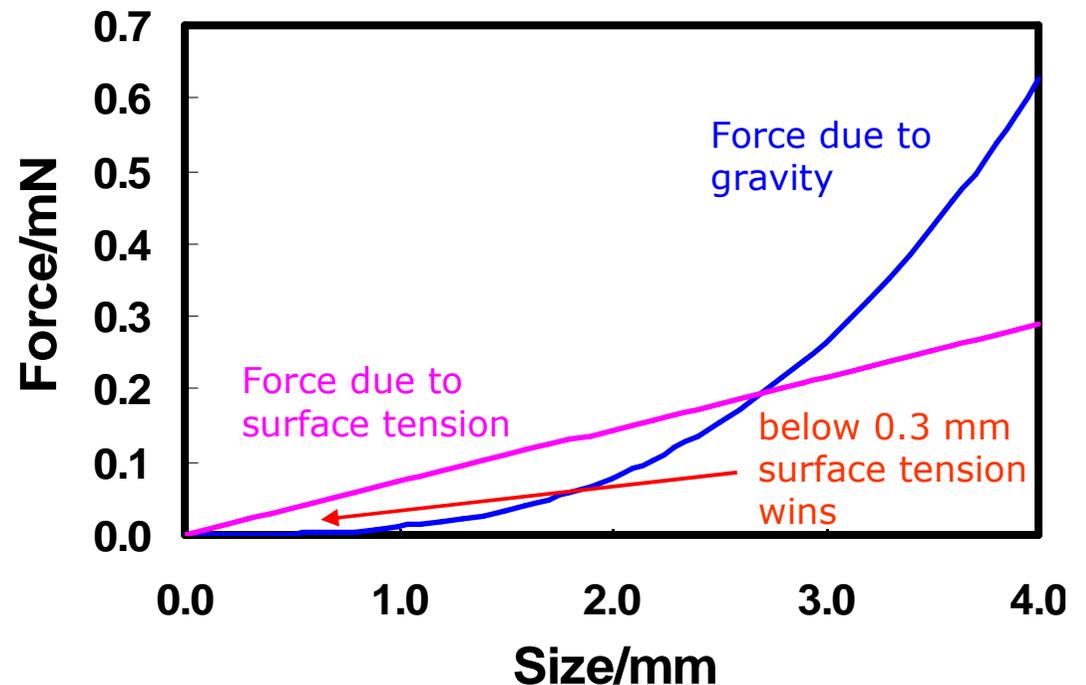
Gravity forces \propto length³

e.g. Force $\sim R^3\rho g$

Small size \Rightarrow surface tension wins

Small means \ll capillary length

$$\kappa^{-1} = (\gamma_{LV}/\rho g)^{1/2} \sim 2.73 \text{ mm (water)}$$



Size Matters: Fiction or Fact?



The Movie – Antz (1998)

Copyright: DreamWorks Animation (1996)



Courtesy: BigWave Productions



Is it just imagination?
Or could it happen?

Water Repellency: Concepts

Naturally Occurring Surfaces

Plants and Leaves



Honeysuckle, Fat Hen, Tulip, Daffodil, Sew thistle (Milkweed), Aquilegia
Nasturtium, Lady's Mantle, Cabbage/Sprout/Broccoli

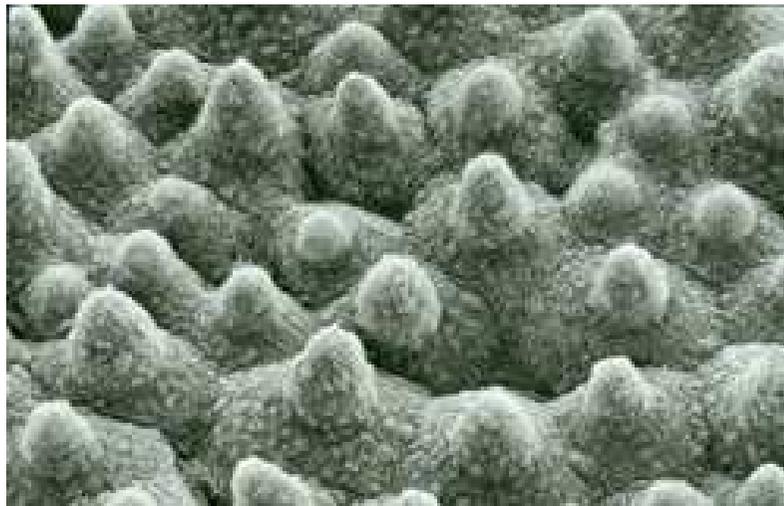
The Sacred Lotus Leaf

Plants

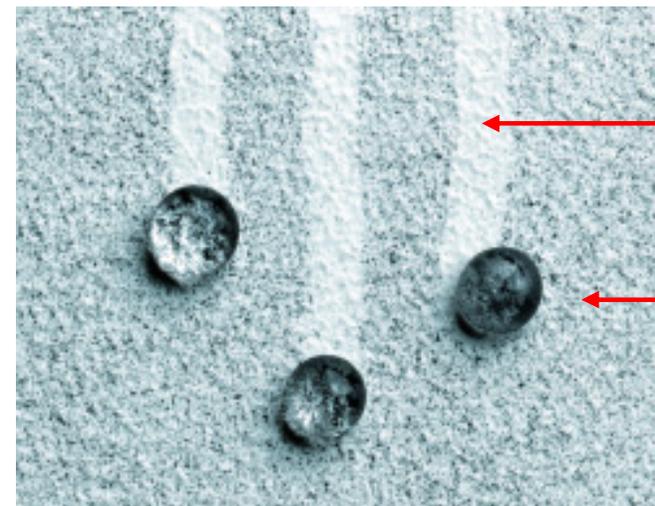
- Many leaves are super-water repellent (i.e. droplets completely ball up and roll off a surface)
- The Lotus plant is known for its purity
- Superhydrophobic leaves are self-cleaning (*under the action of rain*)



SEM of a Lotus Leaf



Self-Cleaning



Dust cleaned away
Dust coated droplet
A “proto-marble”

Acknowledgement Neinhuis and Barthlott

Self-poisoning surface

References Neinhuis, C.; Barthlott, W. *Ann. Bot.*, 79 (1997) 667-677; *Planta* 202 (1997) 1-8.
Onda, T. *et al.*, *Langmuir* 12 (1996) 2125-2127.

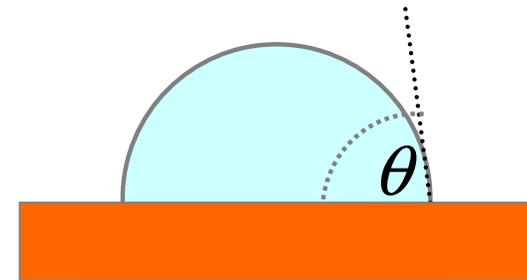
Hydrophobicity and Superhydrophobicity

Surface Chemistry

Terminal group determines whether surface is water-liking or water-fearing
Hydrophobic terminal groups are Fluorine (CF_x) and Methyl (CH_3)

Contact Angles on Teflon

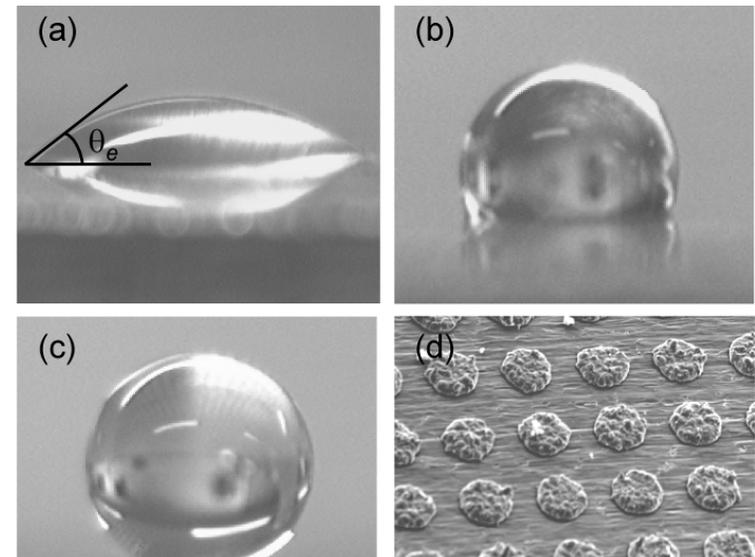
Characterize hydrophobicity
Water-on-Teflon gives $\sim 115^\circ$
The best that *chemistry* can do



Enhancement by Topography

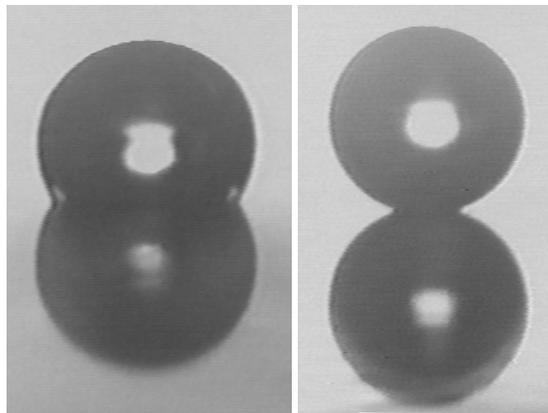
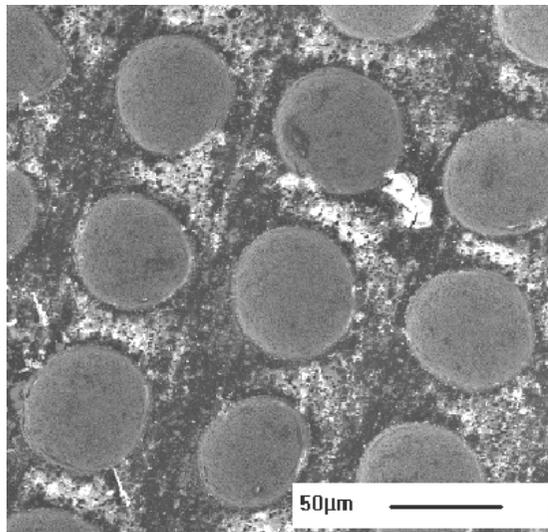
- (a) is water-on-copper
- (b) is water-on-fluorine coated copper
- (c) is a super-hydrophobic surface
- (d) "chocolate-chip-cookie" surface

*Superhydrophobicity is when $\theta > 150^\circ$
and a droplet easily rolls off the surface
(low contact angle hysteresis)*



Superhydrophobicity - Man-Made Examples

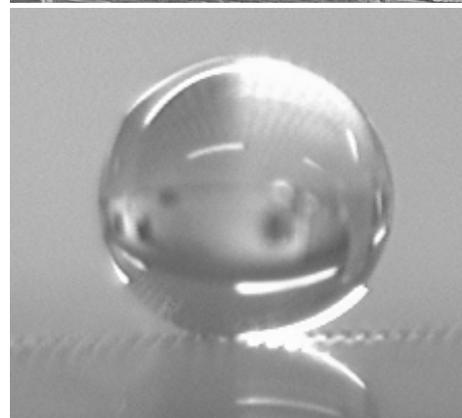
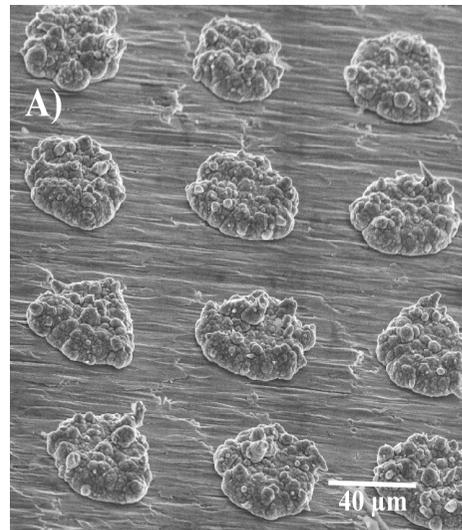
Etched Metal



Flat & hydrophobic

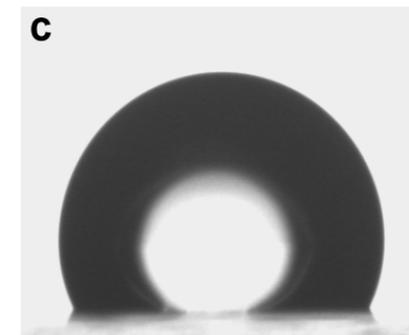
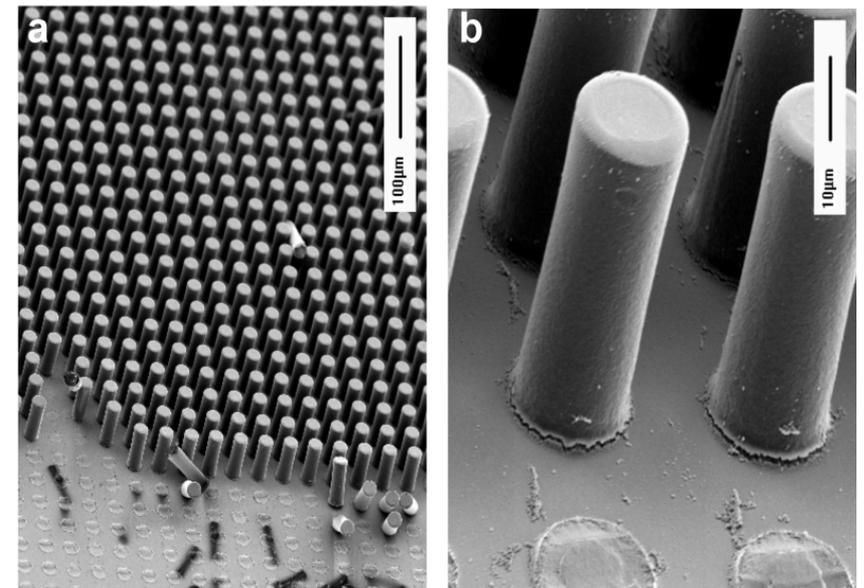
Patterned & hydrophobic

Deposited Metal

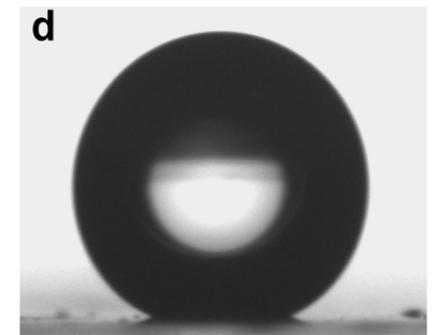


Patterned & hydrophobic

Polymer Microposts



Flat & hydrophobic



Patterned & hydrophobic

References Shirtcliffe, N.J. *et al.*, *Langmuir* **21** (2005) 937-943; *Adv. Maters.* **16** (2004) 1929-1932; *J. Micromech. Microeng.* **14** (2004) 1384-1389.

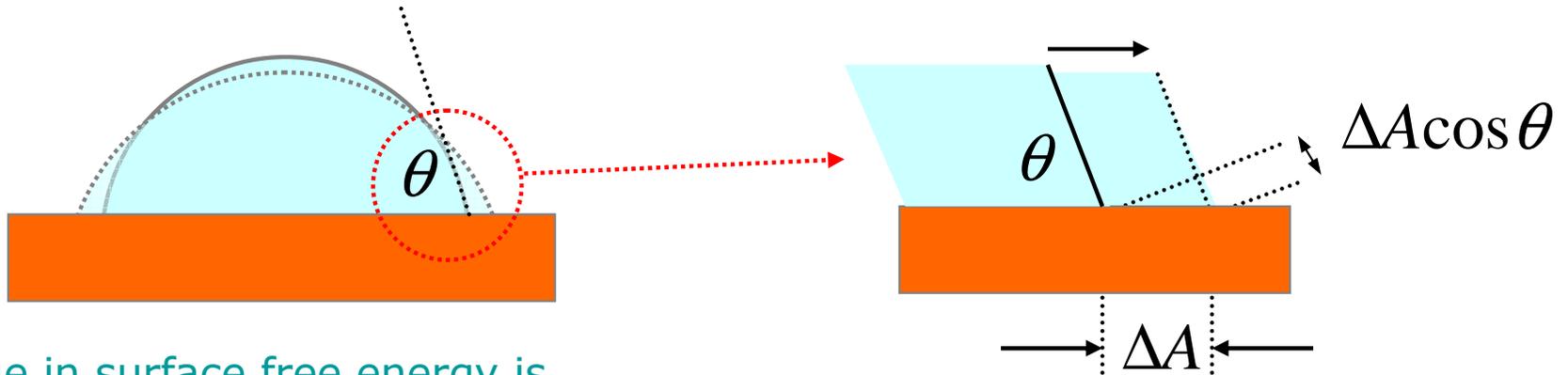
Topography & Wetting: Theory

Surface Free Energy Derivations

Minimum Surface Free Energy

Young's Eq. – The Chemistry

What contact angle does a droplet adopt on a flat surface?



Change in surface free energy is

solid-liquid gain of energy per \times substrate unit area area

solid-vapor loss of energy per \times substrate unit area area

liquid-vapor gain of energy per \times liquid-vapor unit area area

$$\Delta F(x) = (\gamma_{SL} - \gamma_{SV}) \Delta A(x) + \gamma_{LV} \Delta A(x) \cos \theta$$

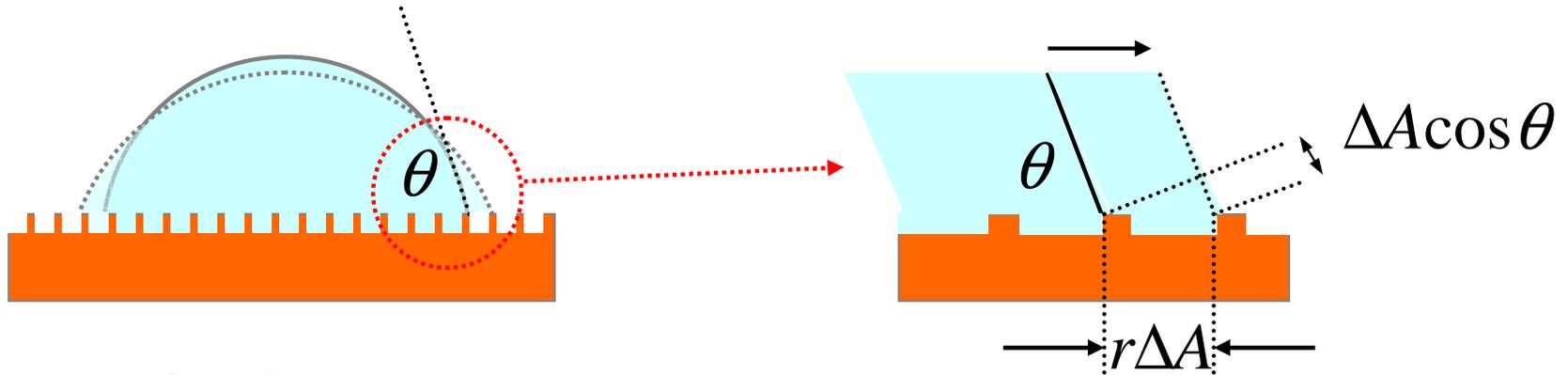
Equilibrium is when $\Delta F(x) = 0 \Rightarrow$

$$\cos \theta_e = (\gamma_{SV} - \gamma_{SL}) / \gamma_{LV}$$

Young's Eq.

Same result as from resolving forces at contact line

Topography 1: Wenzel's Equation



Change in surface free energy is

$$\Delta F(x) = (\gamma_{SL} - \gamma_{SV}) r(x) \Delta A(x) + \gamma_{LV} \Delta A(x) \cos \theta$$

Equilibrium is when $\Delta F(x) = 0 \Rightarrow \cos \theta_W(x) = r(x) (\gamma_{SV} - \gamma_{SL}) / \gamma_{LV}$

$$\cos \theta_W(x) = r(x) \cos \theta_e$$

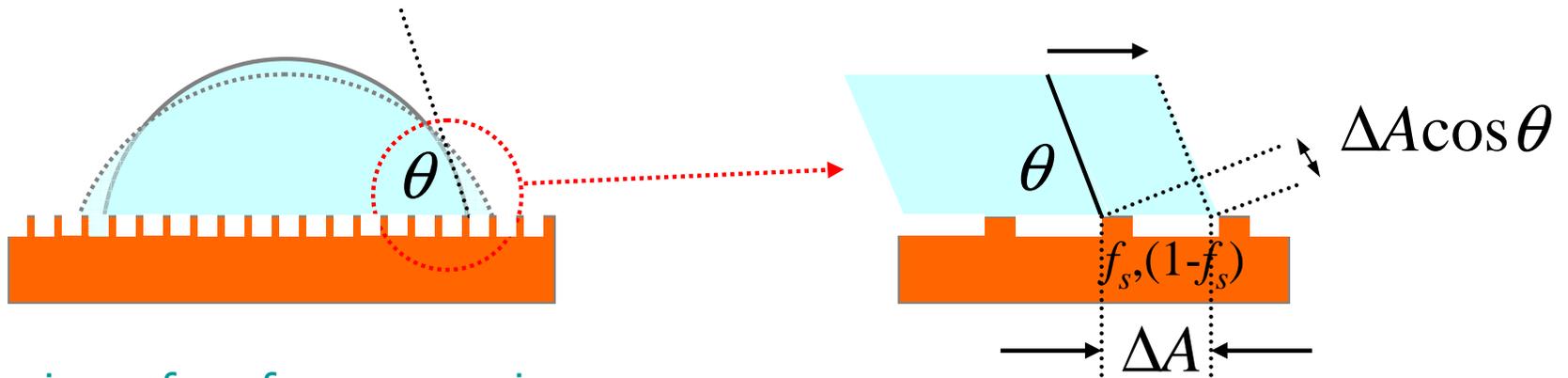
Wenzel Eq

Topography $\Rightarrow r(x)$ = roughness factor

Chemistry \Rightarrow Young's Eq. θ_e

The derivation is based on contact line changes^{\$}, i.e. $r=r(x)$ and $\theta_e(x)$

Topography 2: Cassie-Baxter Equation



Change in surface free energy is

$$\Delta F(x) = (\gamma_{SL} - \gamma_{SV}) f_s(x) \Delta A(x) + \gamma_{LV} (1 - f_s(x)) \Delta A(x) + \gamma_{LV} \Delta A(x) \cos \theta$$

Equilibrium is when $\Delta F(x) = 0 \Rightarrow \cos \theta_{CB}(x) = f_s(x) (\gamma_{SV} - \gamma_{SL}) / \gamma_{LV} - (1 - f_s(x))$

$$\cos \theta_{CB}(x) = f_s(x) \cos \theta_e - (1 - f_s(x))$$

Cassie-Baxter Eq

Topography $\Rightarrow f_s(x) =$ solid surface fraction

Chemistry \Rightarrow Young's Eq. θ_e

Air gaps $\Rightarrow \cos(180^\circ) = -1$

Simplistic view: Weighted average using $f_1(x) \cos \theta_1(x)$ and $f_2(x) \cos \theta_2(x)$

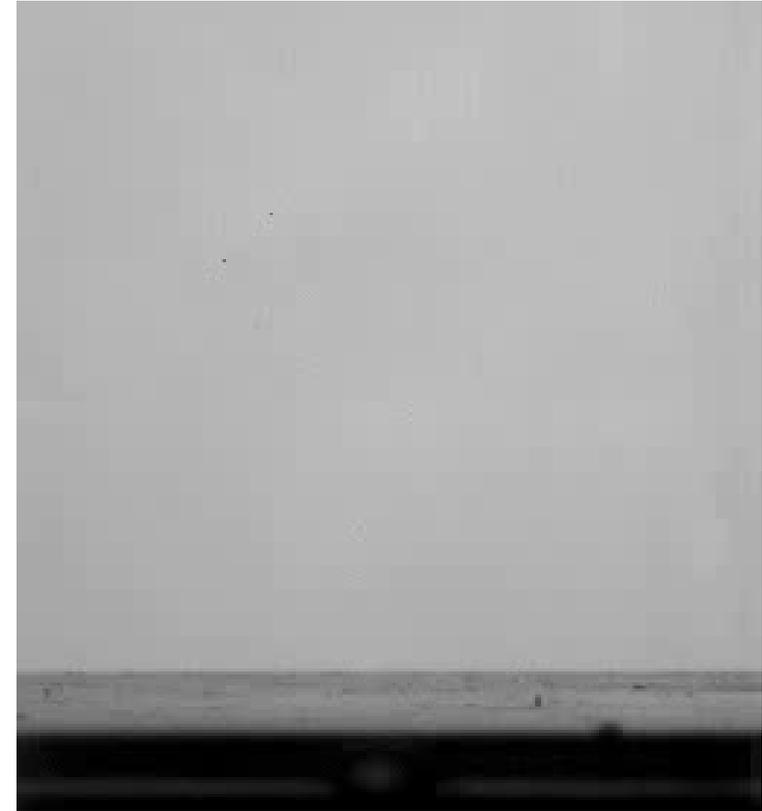
The derivation is based on contact line changes[§], i.e. $f_s = f_s(x)$ and $\theta_e(x)$

References Johnson, R.E.; Dettre, R.H. Adv. in Chem. Series 43 (1964) 112-135. Bico, J.; Marzolin, C.; Quéré, D. Europhys. Lett. 47 (1999) 220-226. §McHale, G. Langmuir 23 (2007) 8200-8205.

Fakir's Carpet (and Bouncing Droplets)



Acknowledgement: Dr Hind & Ms Fretwell (NTU)



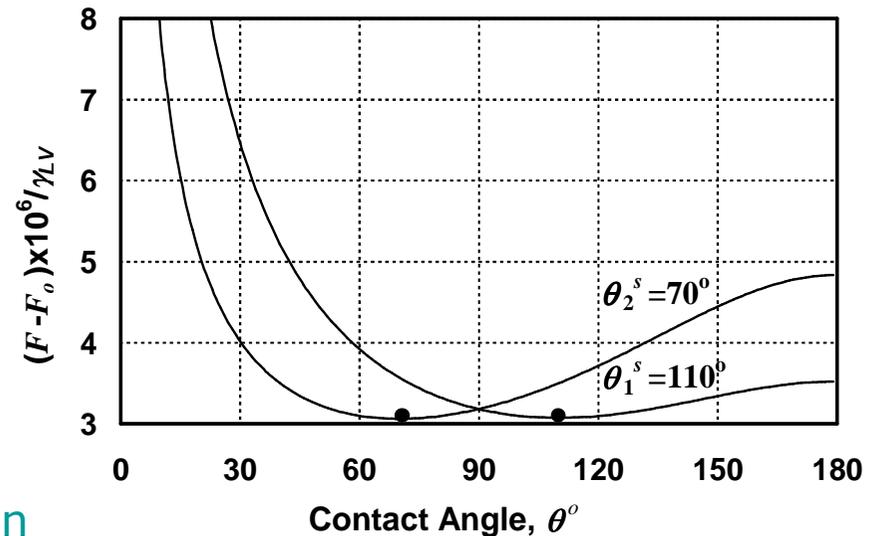
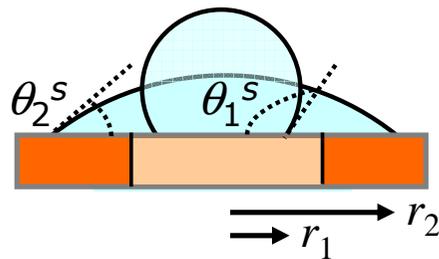
Courtesy: Prof. David Quéré, ESPCI

But liquid skin interacts with solid surfaces and "nails" do not need to be equally separated. A useful analogy, but it is not an exact view.

Importance of the Three-Phase Contact Line

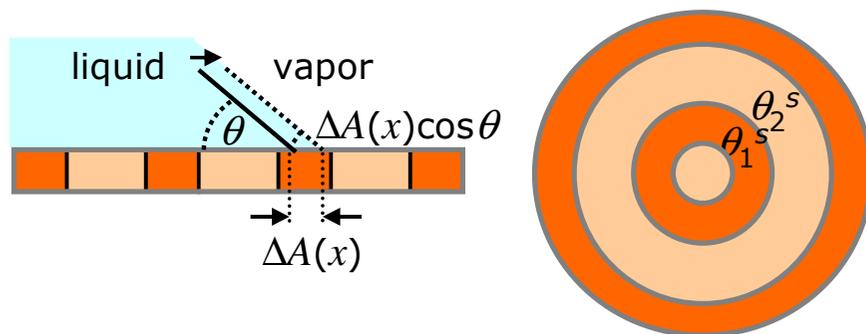
Isolated Defect Surface

Surface has $\theta_1^s = 110^\circ$, $\theta_2^s = 70^\circ$

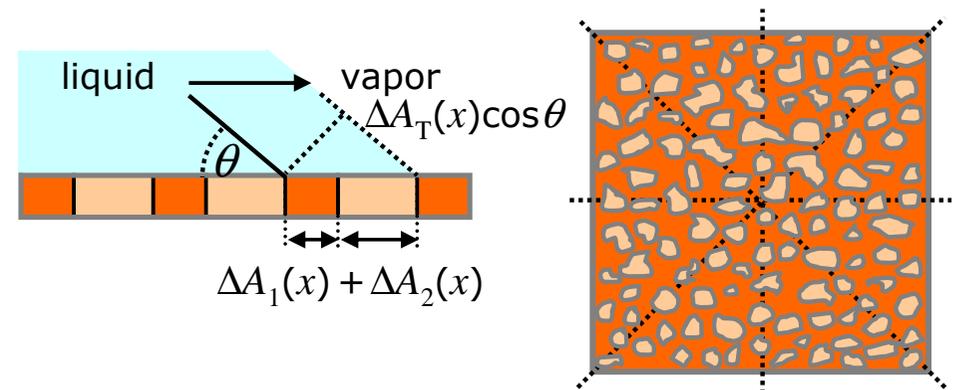


Two droplet configurations exist with min in their local surface free energy corresponding to the same droplet volume

Radial Symmetry



Random Surface



References Gao, L.C.; McCarthy, T.J. Langmuir 23 (2007) 3762-3765. McHale, G. Langmuir 23

06 September 2009 (2007) 8200-8205.

Local and not Global Differential Parameters

Cassie-Baxter

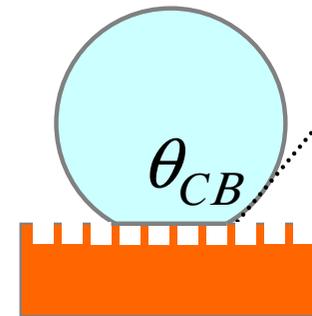
Define surface fractions: $f_i(x) = \Delta A_i(x) / (\Delta A_1(x) + \Delta A_2(x))$

$$\cos \theta_c(x) = f_1(x) \cos \theta_1 + f_2(x) \cos \theta_2$$

for a simple post-type superhydrophobic surface \Rightarrow

$$\cos \theta_{CB}(x) = f_s(x) \cos \theta_e - (1 - f_s(x))$$

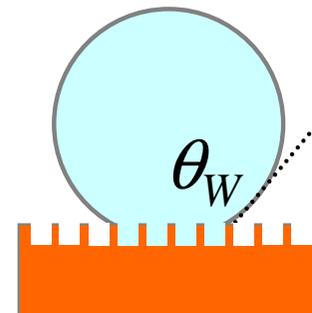
where $f_s(x)$ is the solid surface fraction and the x indicates values at the three-phase contact line ($\theta_e = \theta_e(x)$ is also local to the three-phase contact line)



Wenzel

Define roughness: $r(x) = \Delta A_{wetted}(x) / \Delta A_{projected}(x)$

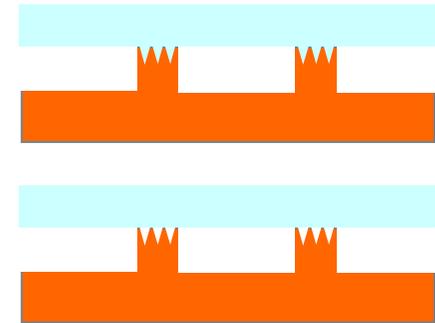
$$\cos \theta_W = r(x) \cos \theta_e$$



Complex Topography

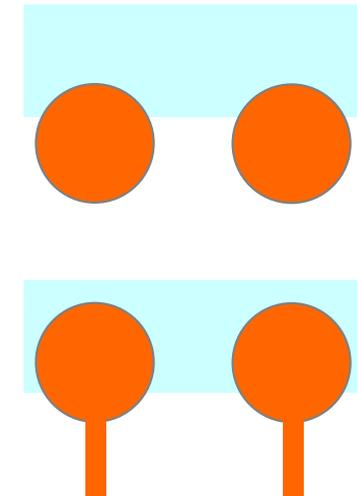
Roughness on Top of Features

- Liquid filled case: Create Wenzel angle and use in Cassie-Baxter equation $\theta_e \rightarrow \theta_w (\theta_e) \rightarrow \theta_{CB} (\theta_w)$
- Non-filled case: Create CB angle for top and use in CB for large scale structure $\theta_e \rightarrow \theta_{CB} (\theta_e) \rightarrow \theta_{CB} (\theta_{CB})$



Curved Features

- Describes fibers¹, spheres and complex shapes
- Recently described as re-entrant shapes²
- Roughness, $r(\theta_e)$, and solid surface fraction, $f_s(\theta_e)$, become dependent on θ_e
- Surfaces can support droplets even when θ_e is substantially below 90°³



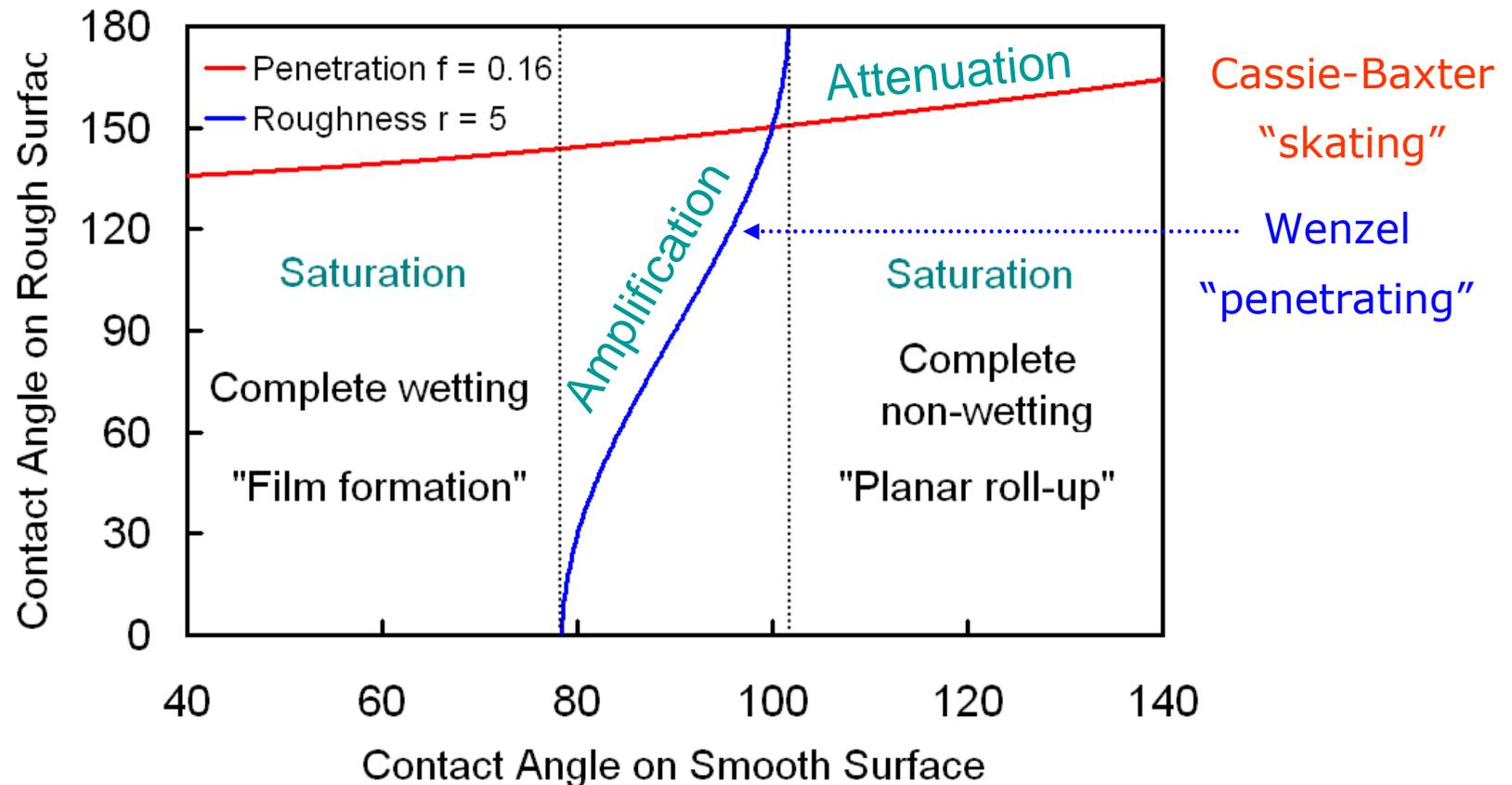
Patterns with Changing Separations

- Roughness, $r(x)$, and solid surface fraction, $f_s(x)$, become dependent on contact line position⁴, x
- Can create gradients in superhydrophobicity⁵

Superhydrophobicity

Consequences

Concepts: Amplification, Attenuation, Saturation



Roughness/Topography

$\theta_e^s > \text{threshold} \Rightarrow \text{enhances repellence}$

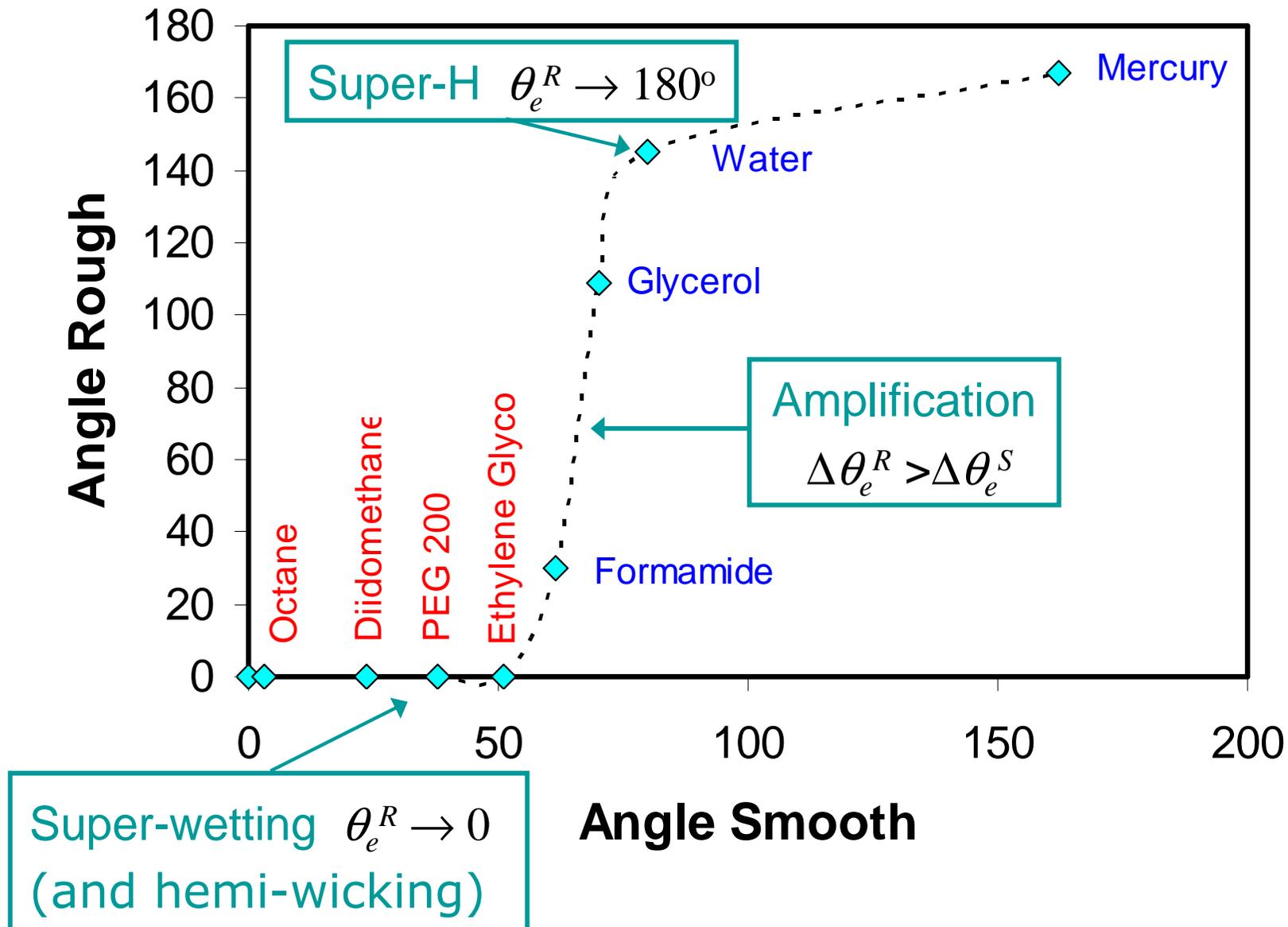
$\theta_e^s < \text{threshold} \Rightarrow \text{enhances film formation}$

Superhydrophobic

"Skating case" \Rightarrow most existing examples

Pressure \Rightarrow transition to penetrating

Liquids on a Superhydrophobic Surface



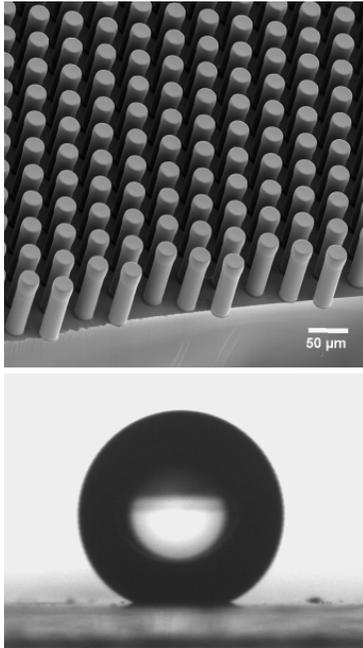
References: McHale, G. *et al.*, *Analyst* **129** (2004) 284-287; *Langmuir* **20** (2004) 10146-10149.

Skating-to-Penetrating Transition

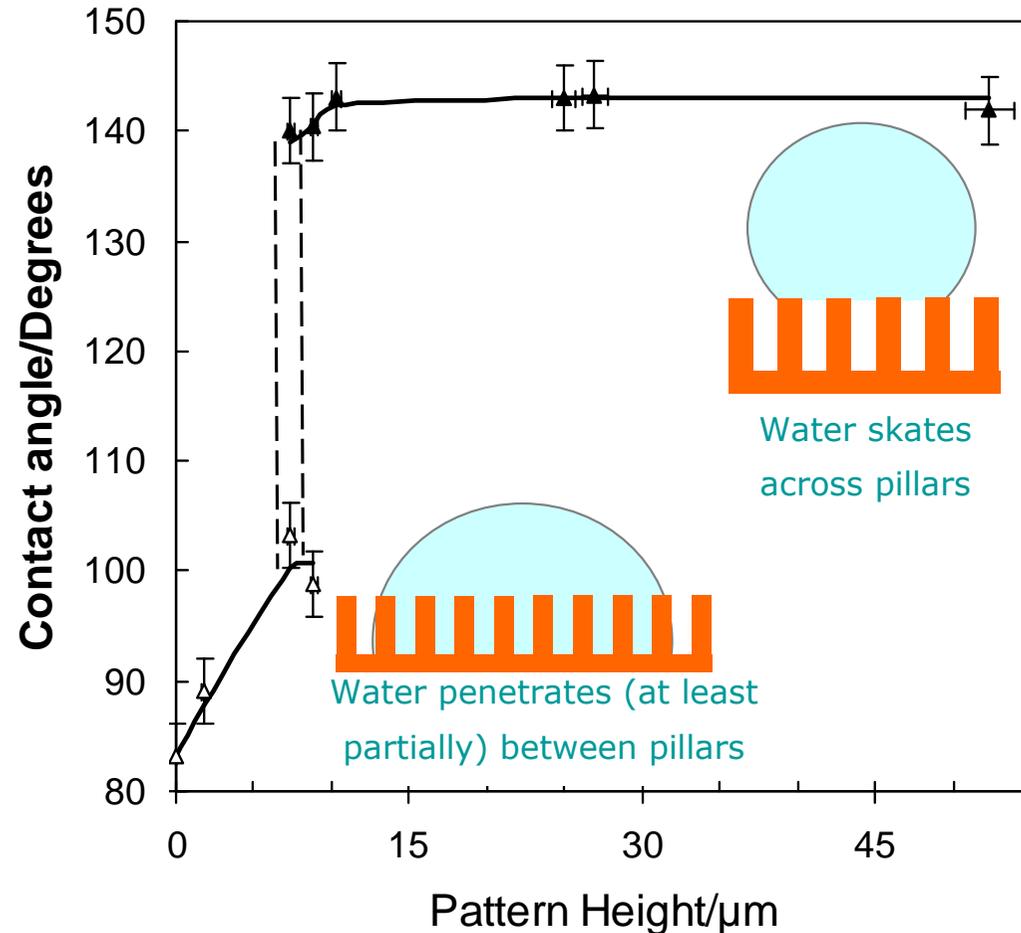
Micro-Structured Surface

SU-8 pillars¹ 15 μm

Hydrophobic treatment



Change of Pillar Height



Quéré Condition

Skating-to-penetrating transition is favoured by surface free energy

considerations when $\theta_W < \theta_{CB}$ (transition may not occur due to sharp features).

Superhydrophobicity and Hysteresis in θ

Experimental Observations of Contact Angle Hysteresis

- Smaller than on flat for the skating (Cassie-Baxter) state
"Slippy" state¹
- Larger than on flat for the penetrating (Wenzel) state
"Sticky" state¹

Models?

- Different views exist possible factors to be considered include:
Shape of tops of features, contact line length², contact area³ (at perimeter)

Gain and Attenuation View

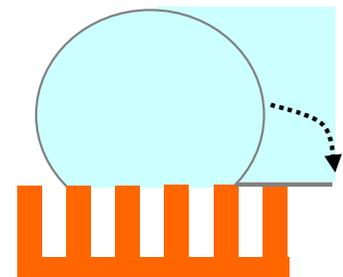
Use CB or W model for θ_A and θ_R
Can work out analytical formulae³
Assumes contact area changes are dominant effect and amplify an intrinsic hysteresis of the surface

2-D Theory World View

CB: To advance must touch next shape and to recede can retract across features⁴

$$\theta_A = 180^\circ \text{ (and } \theta_R = \theta_e^\circ \text{)}$$

3-D world is more complex

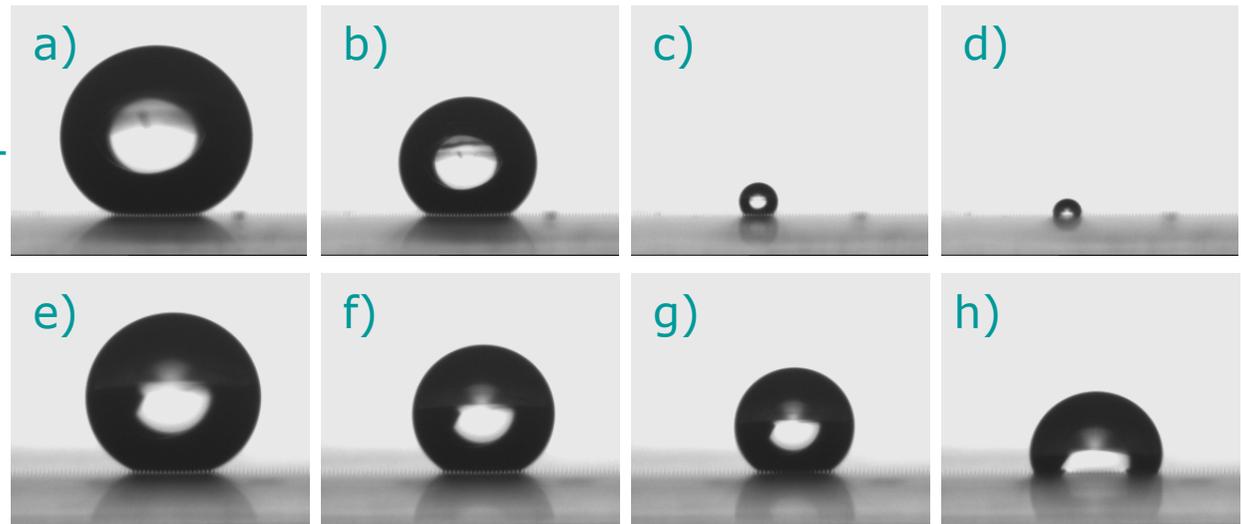


Evaporation and Droplet Collapse

Experiments

Panels a)-d) Late stage collapse from the Cassie-Baxter state. Abrupt/rapid change.¹

Panels e)-h) Mid-stage collapse into Wenzel state. Subsequently, contact line is pinned.¹



Theory/Simulation

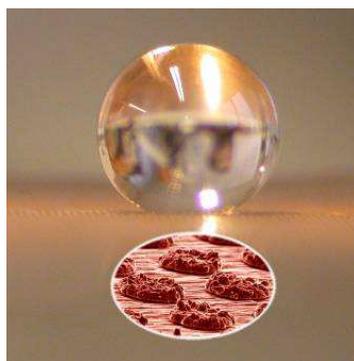
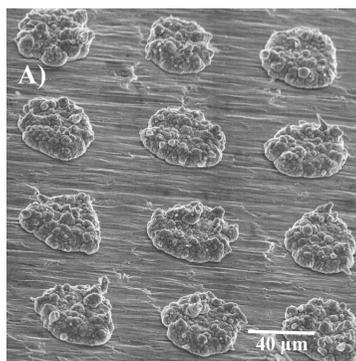
Yeomans² suggests three processes during evaporation:

1. the contact line retreats inwards across the surface
2. the free energy barrier to collapse vanishes and the drop moves smoothly down the posts (long posts)
3. the drop touches the base of the surface patterning and immediately collapses (short posts) – critical curvature of droplet $\propto b^2/h$, where b =gap width and h =post height

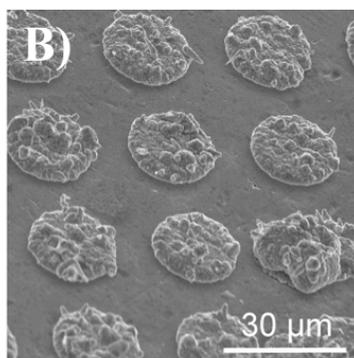
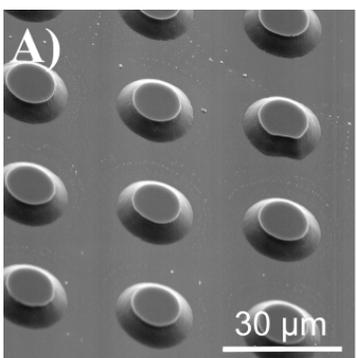
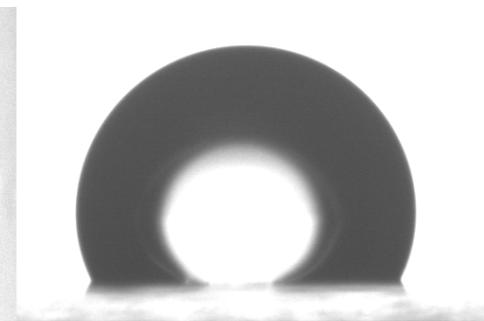
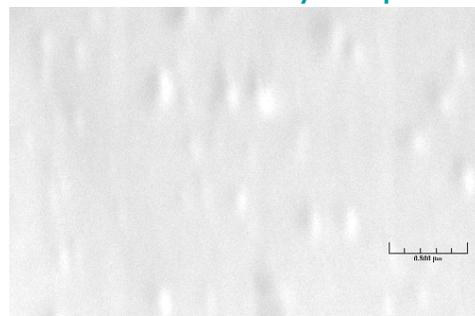
3D simulation suggests the drop can depin from all but the peripheral posts, so that its base resembles an inverted bowl.

References ¹McHale, G. *et al.*, *Langmuir* 21 (2005) 11053–11060. ²Kusumaatmaja, H. *et al.*, *Euro. Phys. Lett.* 81 (2008) art. 36003. Reyssat, M.; Yeomans, J.M.; Quéré, D. *Euro. Phys. Lett.* 81 (2008) art. 26006. Moulinet, S.; Bartolo, D. *Euro. Phys. J.* E24 (2007) 251-260. Bartolo D., *et al.*, *Europhys. Lett.* 74 (2006) 299-305.

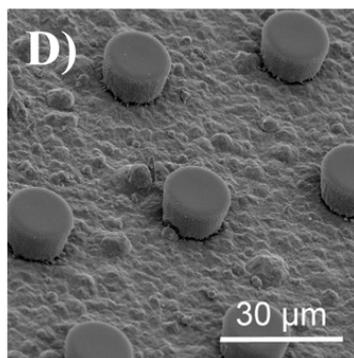
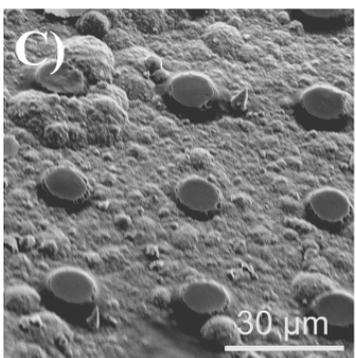
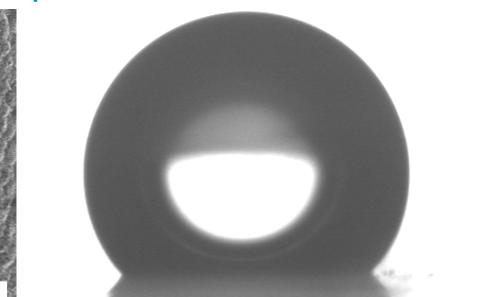
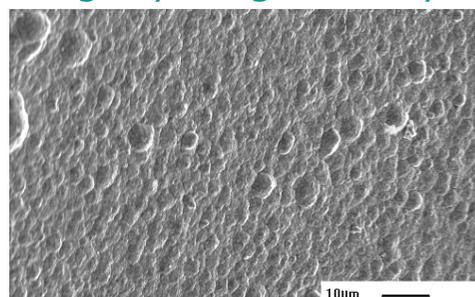
Double Length Scale Systems



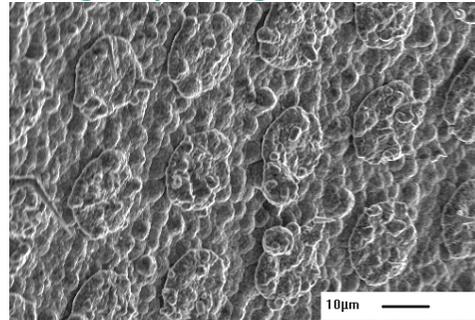
Two length scales is extremely effective
Smooth and hydrophobised: 115°



Slightly rough and hydrophobised: 136°



Slightly rough, textured and hydrophobised: 160°



References Shirtcliffe, N.J. *et al.*, *Adv. Maters.* **16** (2004) 1929-1932; see also: Patankar, N.A.

06 September 2009 Langmuir **20** (2004) 8209-8213.

Path Definition & Self-Actuated Motion

Gradients in Contact Angle

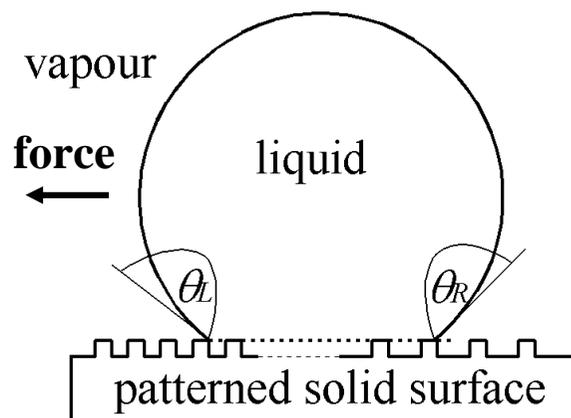
Make contact angle depend on position and surface chemistry $\theta(x, \theta_e^s)$

Same surface chemistry, but vary Cassie-Baxter fraction across surface

$$\cos \theta_{CB}(x) = f(x) \cos \theta_e^s - (1-f(x))$$

Idea

Droplet experiences different contact angles



$$\begin{aligned} \text{Driving force} &\sim \gamma_{LV}(\cos \theta_R - \cos \theta_L) \\ &\sim \gamma_{LV}(f_R - f_L)(\cos \theta_e + 1) \end{aligned}$$

Experiment

Radial gradient $\theta(r) = 110^\circ \rightarrow 160^\circ$



Electrodeposited copper – fractal to overcome hysteresis

References McHale, G. *et al.*, *Analyst* **129** (2004) 284-287; *Langmuir* **20** (2004) 10146-10149.

06 September 2009 McHale, G. *et al.*, in 'Contact Angle, Wettability and Adhesion' Koninklijke Brill NV, vol. **6**, 219-233.; Quéré, D. *Europhys. Lett.* (2009).

Superhydrophobicity

Unexpected superhydrophobicity?

Leidenfrost Effect

Perfect Superhydrophobicity?

Cassie-Baxter with solid fraction $f_s=0$

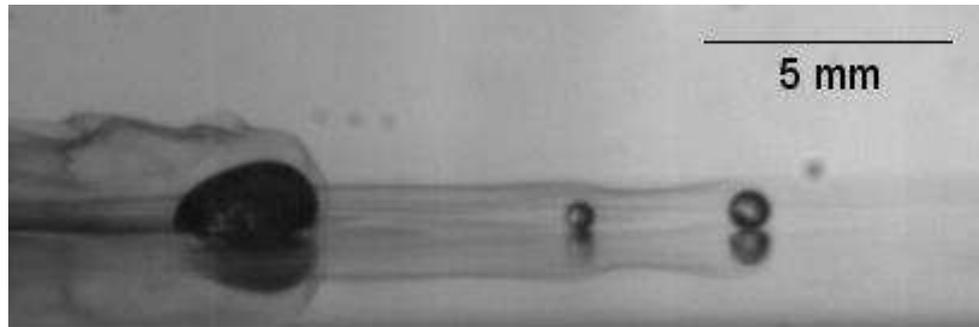
Droplet floats on a layer of vapor: $\cos\theta_{CB}=0 \times \cos\theta_e - (1-0) \Rightarrow \theta_{CB}=180^\circ$

Droplet of water deposited onto a hot surface ($\sim 200^\circ\text{C}$)

Thin vapor layer forms and insulates rest of droplet (only slowly evaporates)

Droplet is completely non-wetting and mobile

Leidenfrost Droplets



Liquid nitrogen poured on water at ambient temperature slides on an "air cushion" over the liquid surface

Leidenfrost Puddle



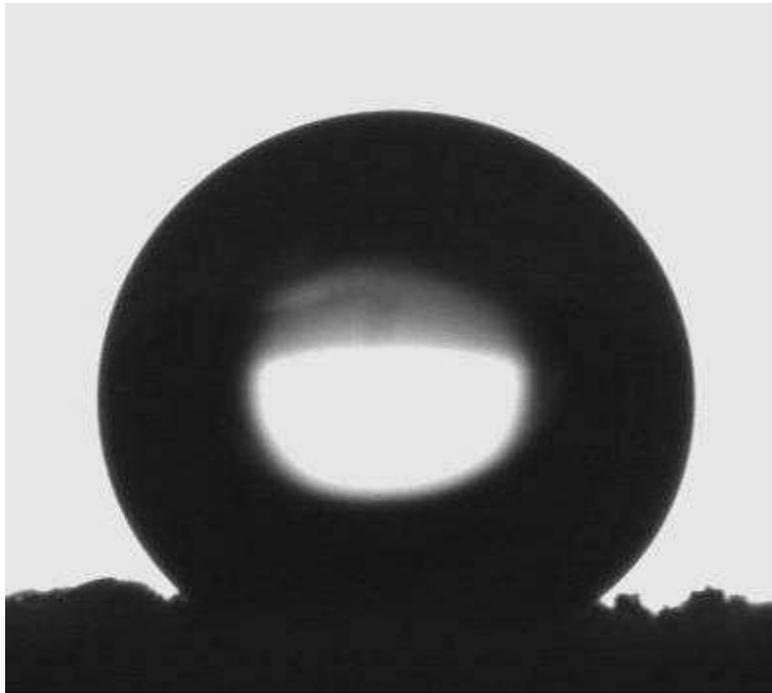
FIG. 2. Large water droplet deposited on a silicon surface at 200°C .

Reference Biance, A.L.; Clanet, C.; Quéré, D. Phys. Fluids. 15 (2003) 1632-1637.

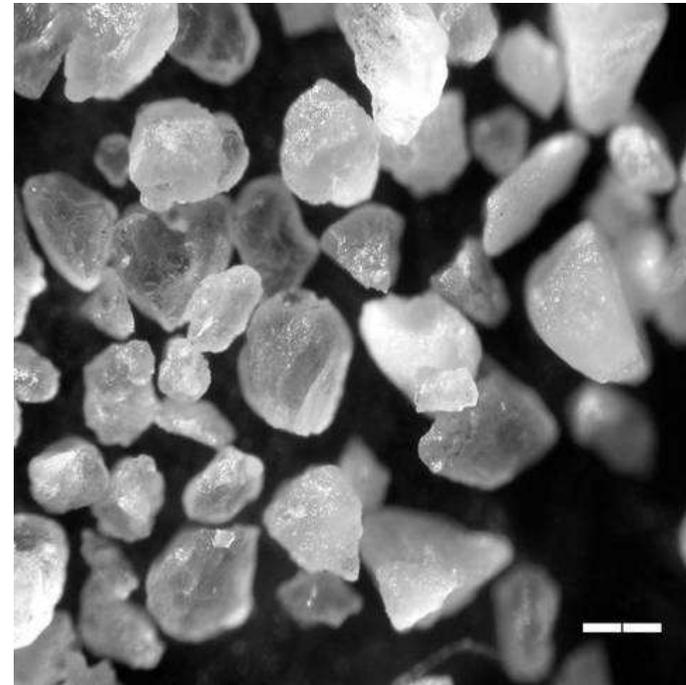
06 September 2009 <http://www.pmmh.espci.fr/fr/gouttes/recherche/>

Super Water-Repellent Sand/Soil

Sand with 139°



Shape and Packing



↔
200 μm

Comments

1. Effect occurs naturally, but can also be reproduced in the lab
2. Water droplet doesn't penetrate, it just evaporates
3. Need to use ethanol rich mixture to get droplet to infiltrate (MED test)

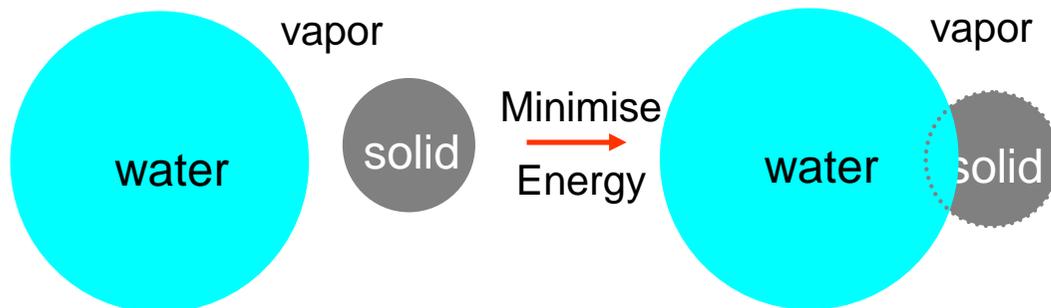
References McHale, G. *et al.*, *Eur. J. Soil Sci.* 56 (2005) 445-452; McHale, G. *et al.*, *Hydrol. Proc.* 21 (2007) 2229-2238. Bachmann, J. & McHale, G. *Eur. J. Soil Sci.* 60 (2009) 420-430.

Liquid Marbles

Loose Surfaces

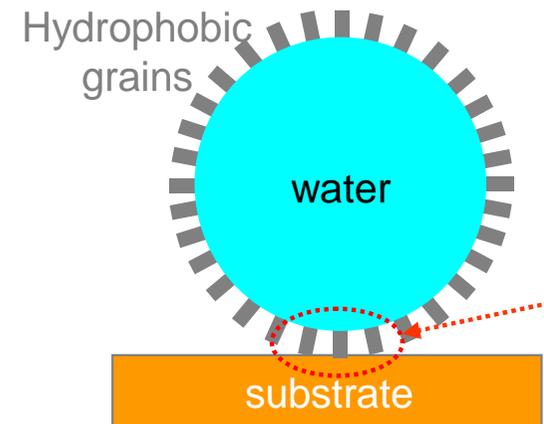
1. Loose sandy soil – grains are not fixed, but can be lifted
2. Surface free energy favors solid grains attaching to liquid-vapor interface
3. A water droplet rolling on a hydrophobic sandy surface becomes coated and forms a liquid marble

Hydrophobic Grains and Water



$$\Delta F = -\pi R_g^2 \gamma_{LV} (1 + \cos \theta_e)^2$$

Energy is always reduced on grain attachment



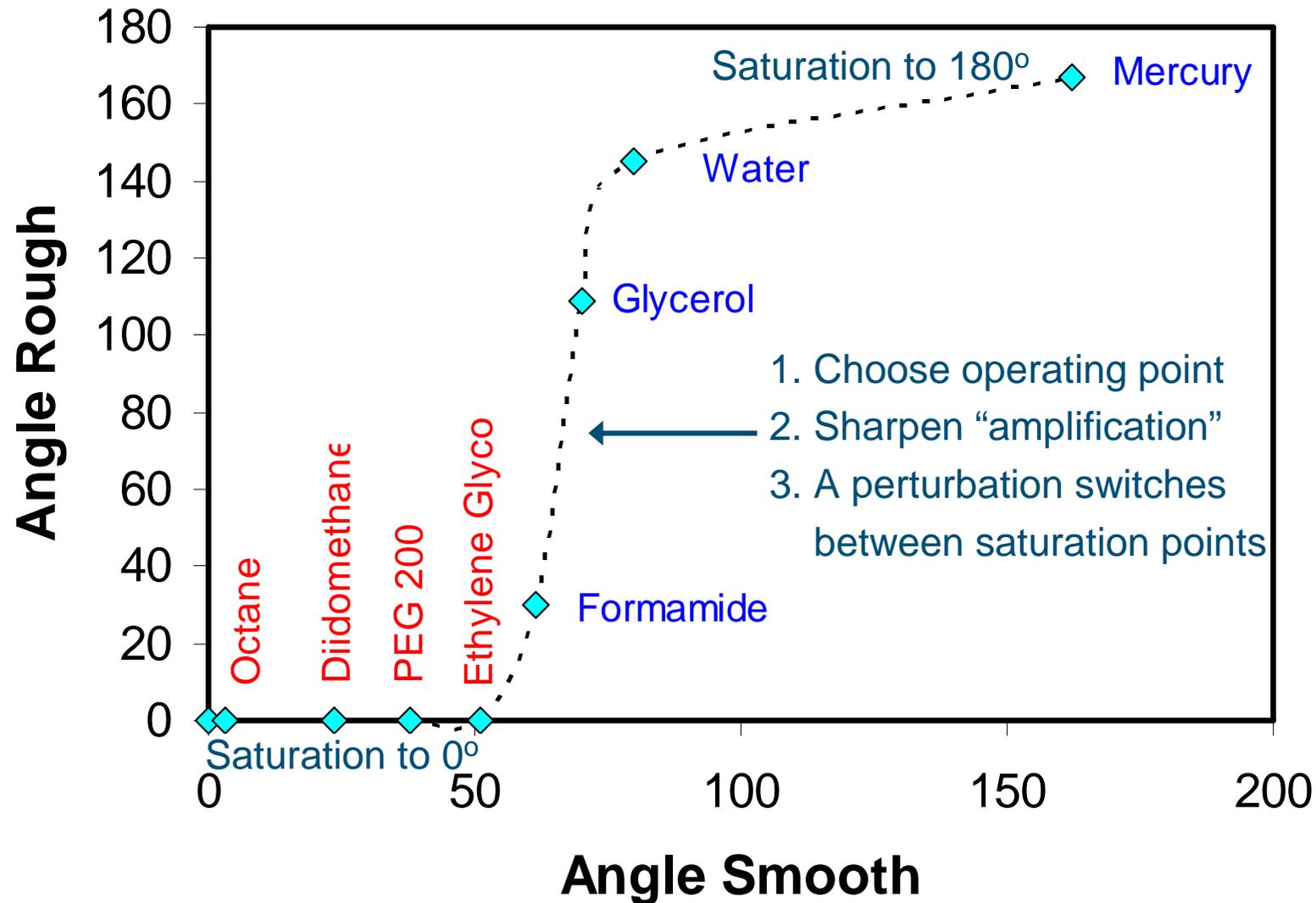
Similar to pillars, but solid conformable to liquid



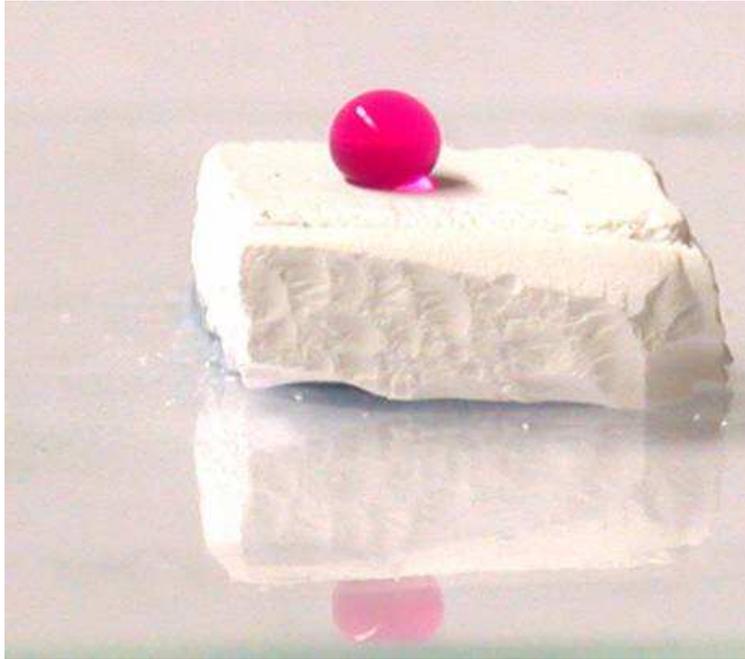
Porosity, Spreading and Imbibition

*Superwetting, Superspreading,
Hemi-wicking and Porosity*

Digital Switching



Sol-Gel: Switching off Superhydrophobicity



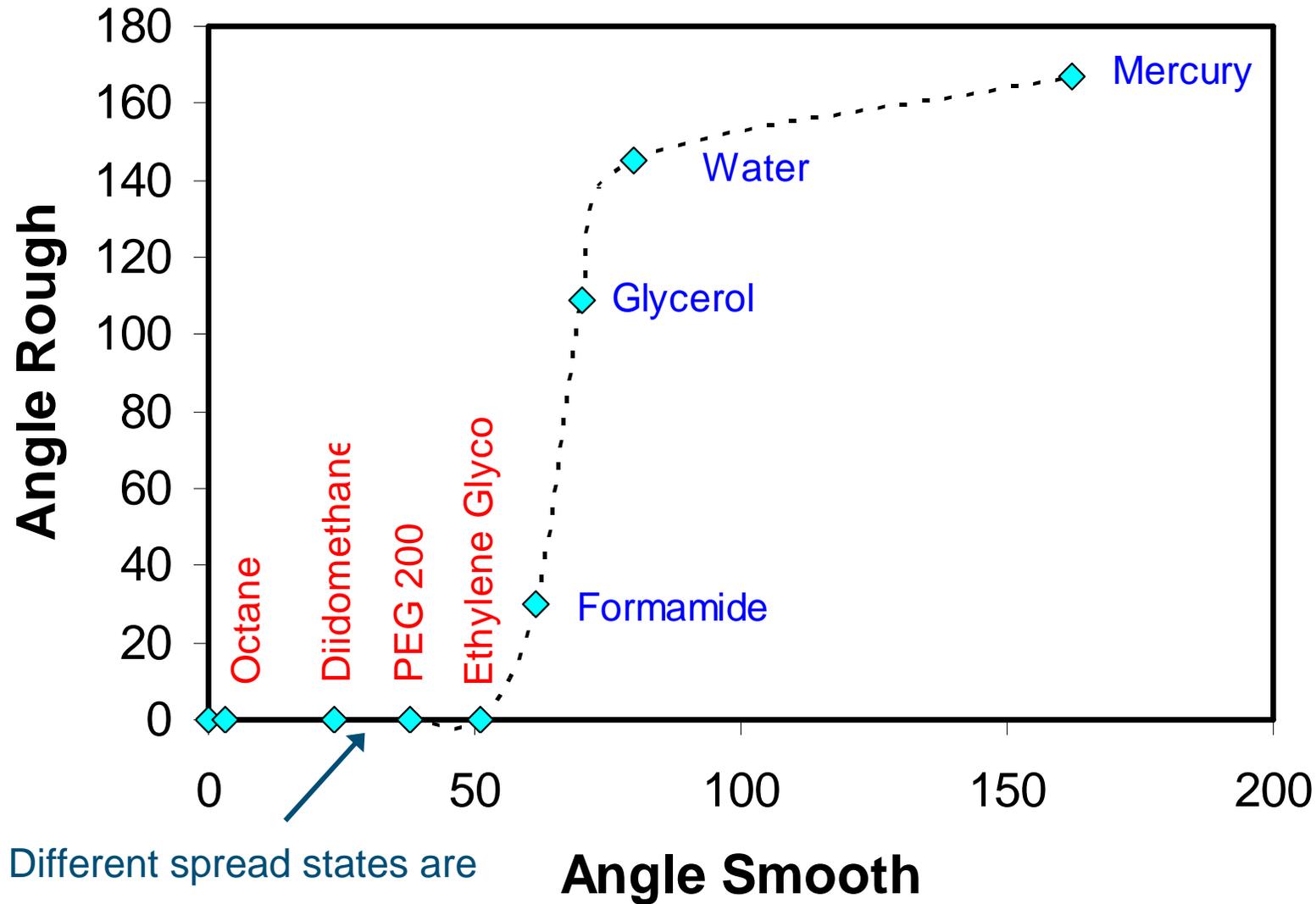
→
Foam heated
(and cooled)
prior to droplet
deposition

Mechanisms for Switching

- Temperature history of substrate
- Surface tension changes in liquid (alcohol content, surfactant, ...)
- Electrowetting

Switch could trigger a large change ⇒ Sensor based on hydrophobicity

Super-spreading



Different spread states are approached at different rates

References McHale, G. *et al.*, *Analyst* **129** (2004) 284-287; *Phys. Rev. Lett.* **93** (2004) art. 036102.

Driving Forces for Spreading

Drop spreads radially until contact angle reaches equilibrium

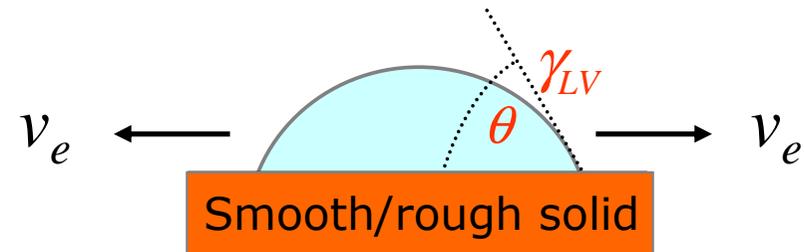
Horizontally projected force $\gamma_{LV} \cos \theta$

Smooth Surface

Driving force $\sim \gamma_{LV}(\cos \theta_e^s - \cos \theta)$

Cubic drop edge speed

$$\Rightarrow v_E \propto \theta \gamma_{LV}(\theta^2 - \theta_e^{s2})$$



Wenzel Rough Surface

Driving force $\sim \gamma_{LV}(r \cos \theta_e^s - \cos \theta)$

Linear droplet edge speed

$$\Rightarrow v_E \propto \theta \gamma_{LV}((r-1) + ((\theta^2 - r\theta_e^{s2})/2))$$

Prediction : Weak roughness (or surface texture) modifies edge speed:

$$v_E \propto \theta(\theta^2 - \theta_e^{s2}) \quad \text{changes towards} \quad v_E \propto \theta$$

Superspreading of PDMS on Pillars

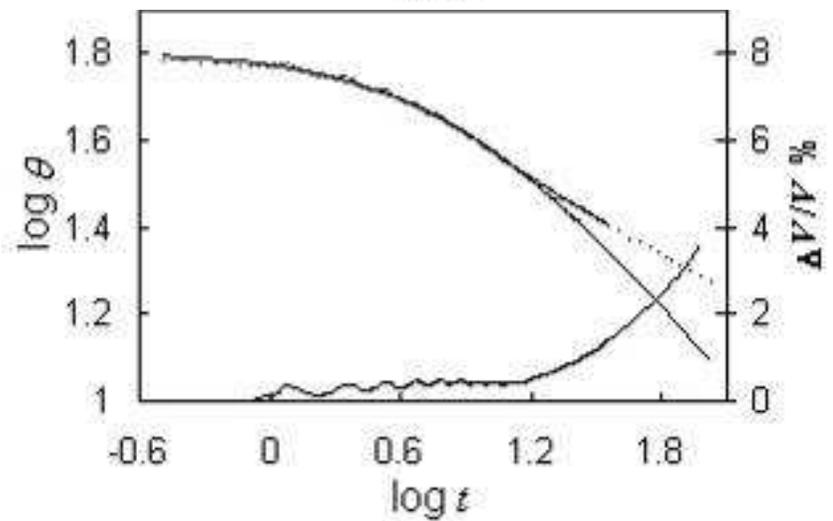
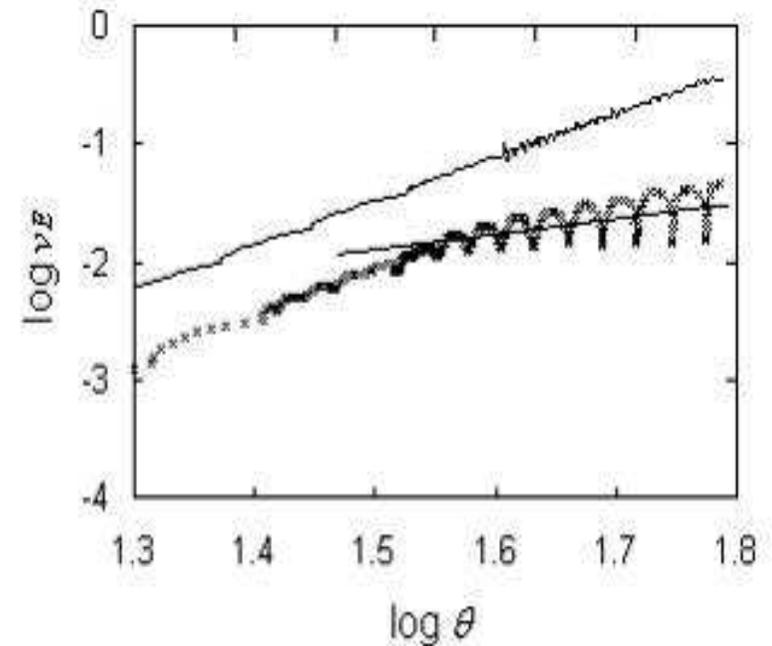
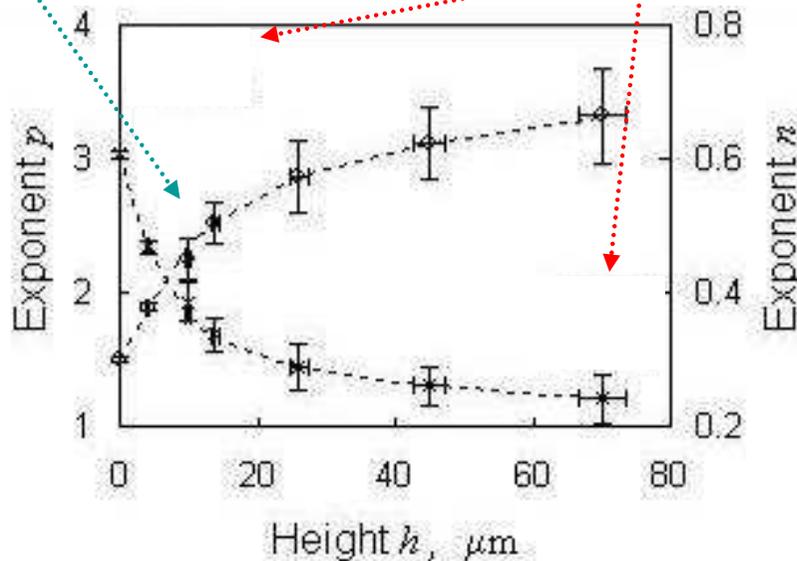
Tanner's Law exponents p and n

(cubic to linear transition)

$$v_E \propto v^* \theta^p \quad \theta \propto \left(\frac{V^{1/3}}{v^*} \right)^n \frac{1}{(t + t_o)^n}$$

Effect of substrate on PDMS

Effect of substrate on water



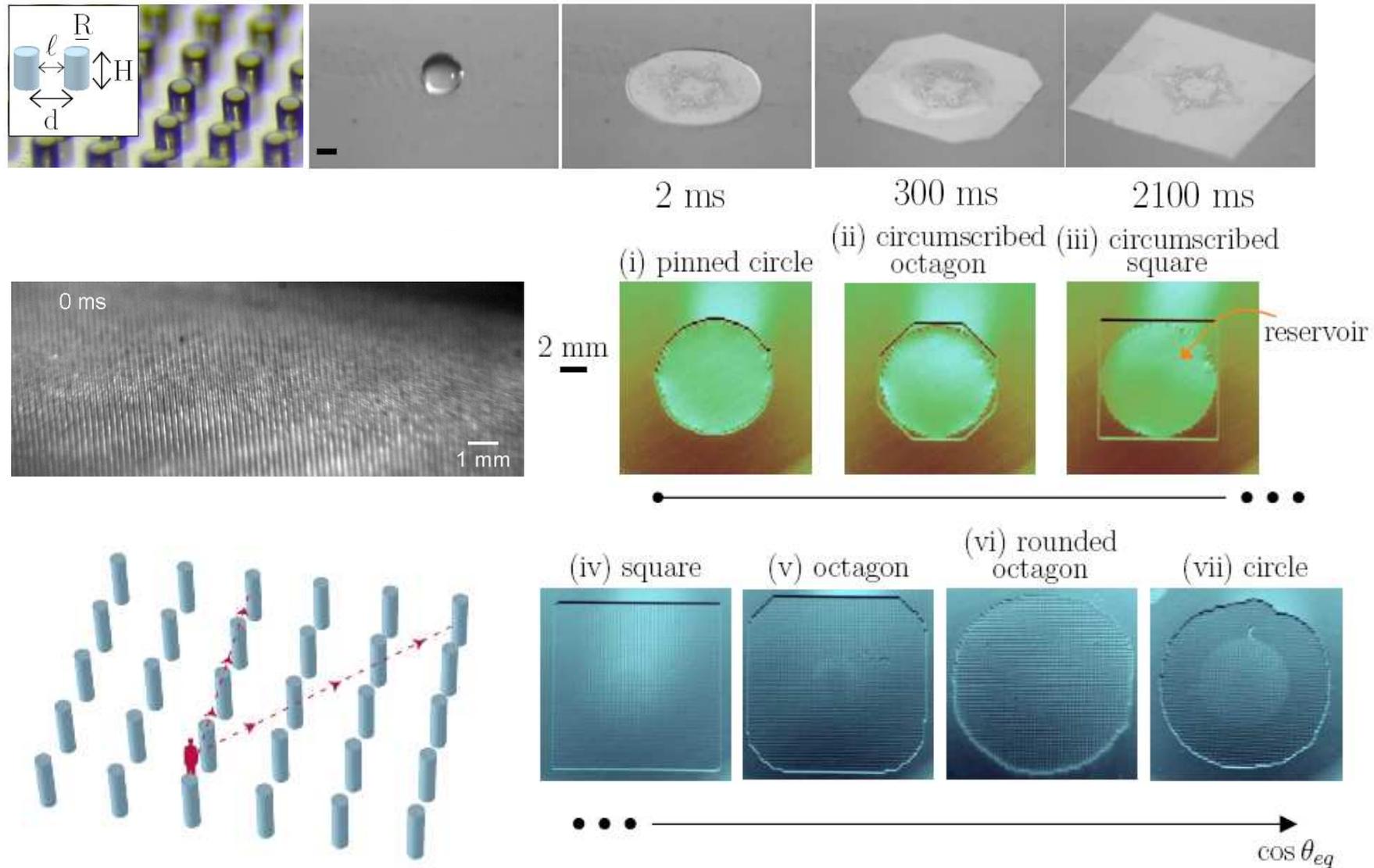
Reference McHale, G. et al., Phys. Rev. Lett. 93 (2004) art. 036102.

06 September 2009

37



Topography Induced Wetting: Hemi-Wicking

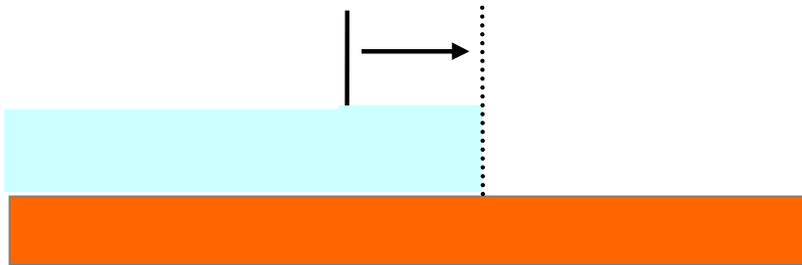


References Bico, J. *et al.*, *Coll. Surf. A* **206** (2002) 41-46. Quéré, D. *Physica A* **313** (2002) 32-46.

06 September 2009 Courbin *et al.*, *Nature Materials*. **6** (2007) 661-664; McHale, **6** (2007) 627-638.

Hemi-Wicking: Theory

Flat Surface



Change in surface free energy is

$$\Delta F = (\gamma_{SL} - \gamma_{SV})\Delta A + \gamma_{LV} \Delta A$$

liquid is assumed to be infinitesimally thin

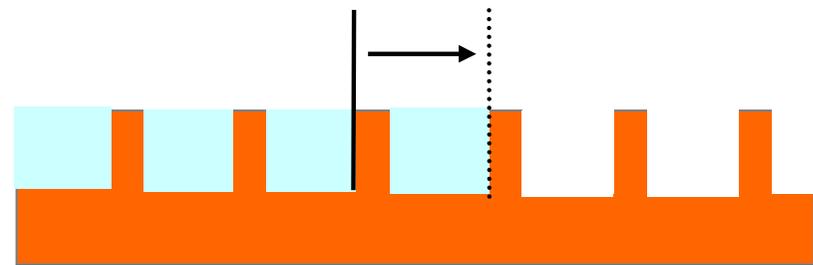
Spreading is when $\Delta F < 0 \Rightarrow$

$$\boxed{(\gamma_{SL} - \gamma_{SV}) / \gamma_{LV} > 1}$$

i.e. critical angle is

$$\cos \theta_c = (\gamma_{SL} - \gamma_{SV}) / \gamma_{LV} = 1 \Rightarrow \theta_e = 0^\circ$$

Textured Surface



Change in surface free energy is

$$\Delta F = (\gamma_{SL} - \gamma_{SV}) (r - f_s) \Delta A + \gamma_{LV} (1 - f_s) \Delta A$$

extra surface area excluding tops of features

Imbibition is when $\Delta F < 0 \Rightarrow$

$$\boxed{\theta_e < \theta_c \text{ where } \cos \theta_c = (1 - f_s) / (r - f_s)}$$

i.e. critical angle is between 0° and 90°

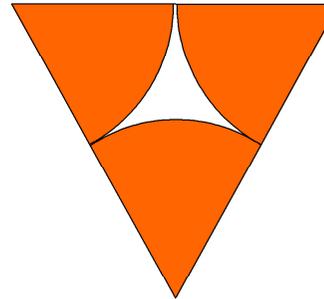
(usual porous media is equivalent to $r \rightarrow \infty$)

Transition from Wetting to Porosity

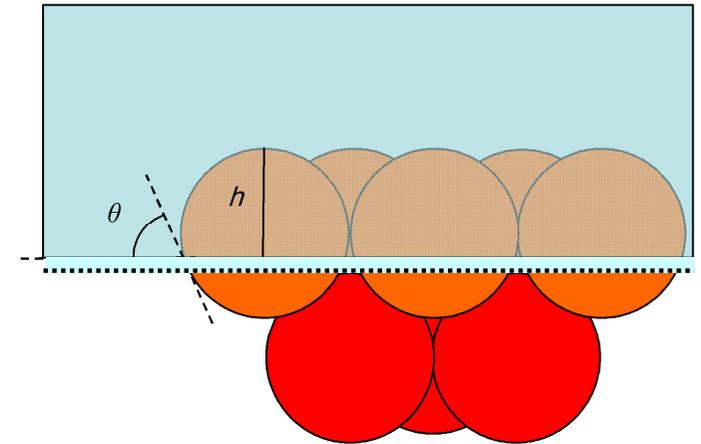
Assumptions

1. Spherical particles radius R
2. Fixed & hexagonally packed
3. Planar meniscus with Young's eq. contact angle, θ_e
4. Minimise surface free energy, F

Top View



Side View



Results for Close Packing

1. Change in surface free energy with penetration depth, h , into first layer of particles
2. Equilibrium exists provided liquid does not touch top particle of second layer
3. If liquid touches second layer at depth, h_c , then complete infiltration is induced
4. Critical contact angle, θ_c , when h_c reached^{1,2}

$$\Delta F = -\pi R \gamma_{LV} \left[\cos \theta_e + \left(1 - \frac{h}{R} \right) \right] \Delta h$$

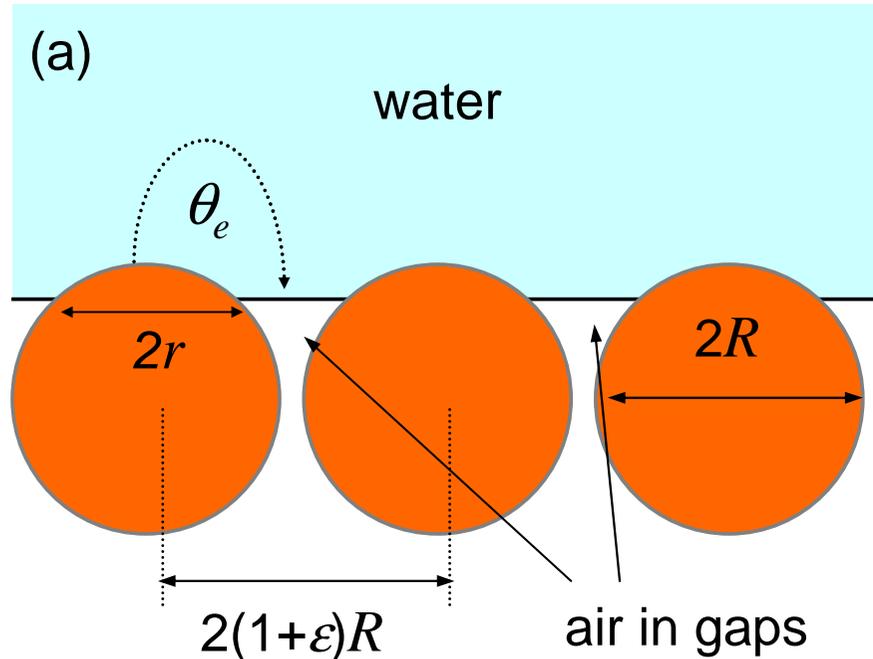
$$h_c = \sqrt{\frac{8}{3}} R = 1.63 R$$

$$\theta_c = 50.73^\circ$$

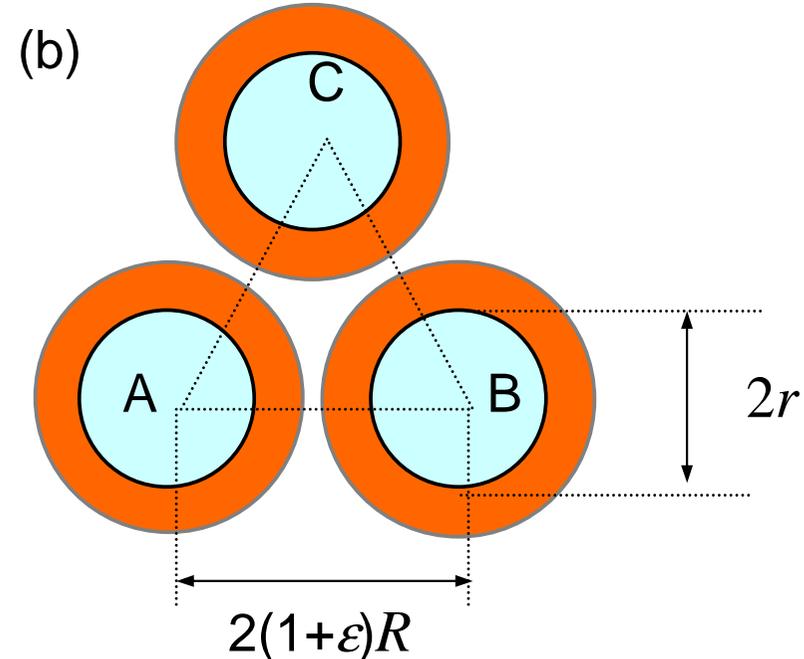
Creating superhydrophobic surfaces with curved features allows liquids to be supported even when $\theta_e < 90^\circ$ – so-called re-entrant surface features³

Model Extreme Water-Repellent Soil

Side View



Top View



Comments

1. Effect occurs naturally (forest fires in sandy areas, grey water usage, ...)
2. Can model using uniform size, smooth spheres in a hexagonal arrangement
2. Water bridges air gaps horizontally between spheres – Mixed Wenzel/CB system

$$\theta_e \xrightarrow{\text{Wenzel}} \theta_W \xrightarrow{\text{Cassie-Baxter}} \theta_{CB}$$

One Last Thought

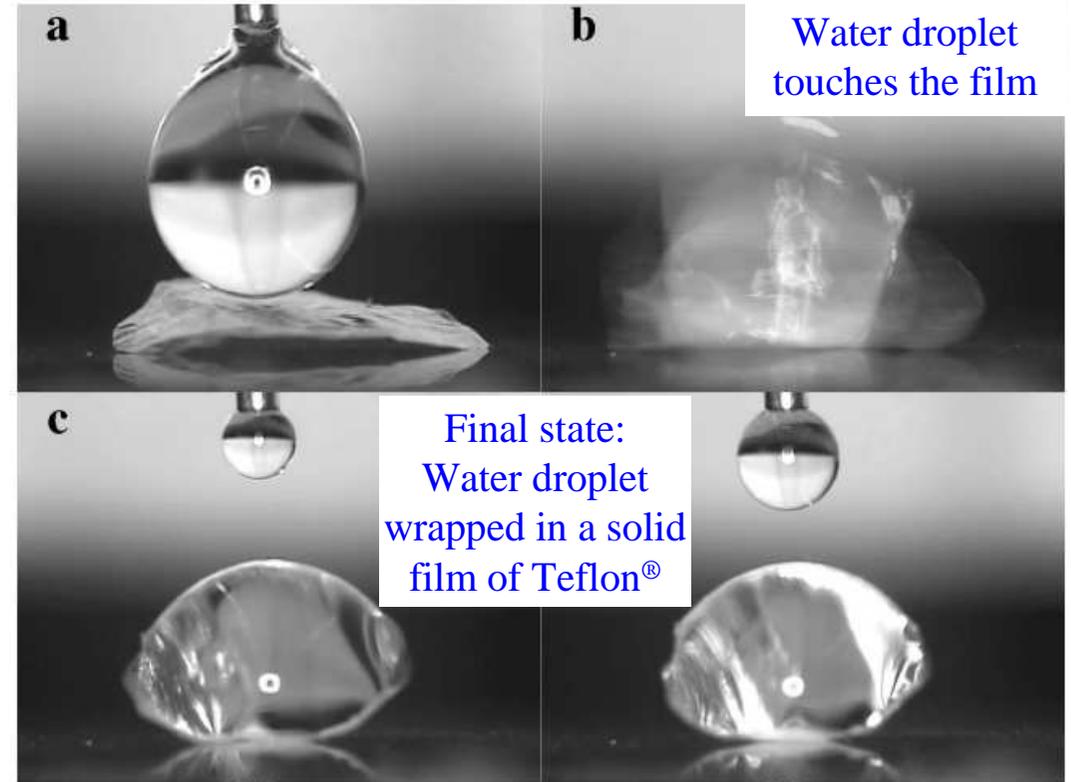
Do Hydrophobic Surfaces Exist?

1. We all know Teflon[®] is a hydrophobic solid and gives a non-stick surface
2. Consider a thin, 3.7 μm , film of Teflon[®] AF2400 contacted by a droplet of water

Droplet Wrapping Video



Stills from Video



Courtesy: Prof. Tom McCarthy (UMass, Amherst)

If a droplet wraps itself up in Teflon[®] ... is this consistent with Teflon[®] being hydrophobic?

Summary

1. Basics of Superhydrophobicity

- Well developed conceptual models
- Often over-simplified use of Cassie-Baxter and Wenzel equations
- Can design applications to take advantage of the effects

2. Beyond Simple Superhydrophobicity

- Many other systems (e.g. soil) can be viewed as superhydrophobic
- Wetting, spreading, wicking and porous systems are of future interest
- Slip, drag reducing and functional properties are now being seriously investigated (See our contributed paper)

The End

Acknowledgements

Collaborators

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PDRA's	Dr Neil Shirtcliffe, Dr Dale Herbertson, Dr Carl Evans, Dr Paul Roach, Dr Yong Zhang
PhD's	Ms Sanaa Aqil, Mr Steve Elliott

Funding Bodies

GR/R02184/01 – Superhydrophobic & superhydrophilic surfaces

GR/S34168/01 – Electrowetting on superhydrophobic surfaces

EP/C509161/1 – Extreme soil water repellence

EP/D500826/1 & EP/E043097/1 – Slip & drag reduction

EP/E063489/1 – Exploiting the solid-liquid interface

Dstl via EPSRC/MOD JGS, Kodak European Research

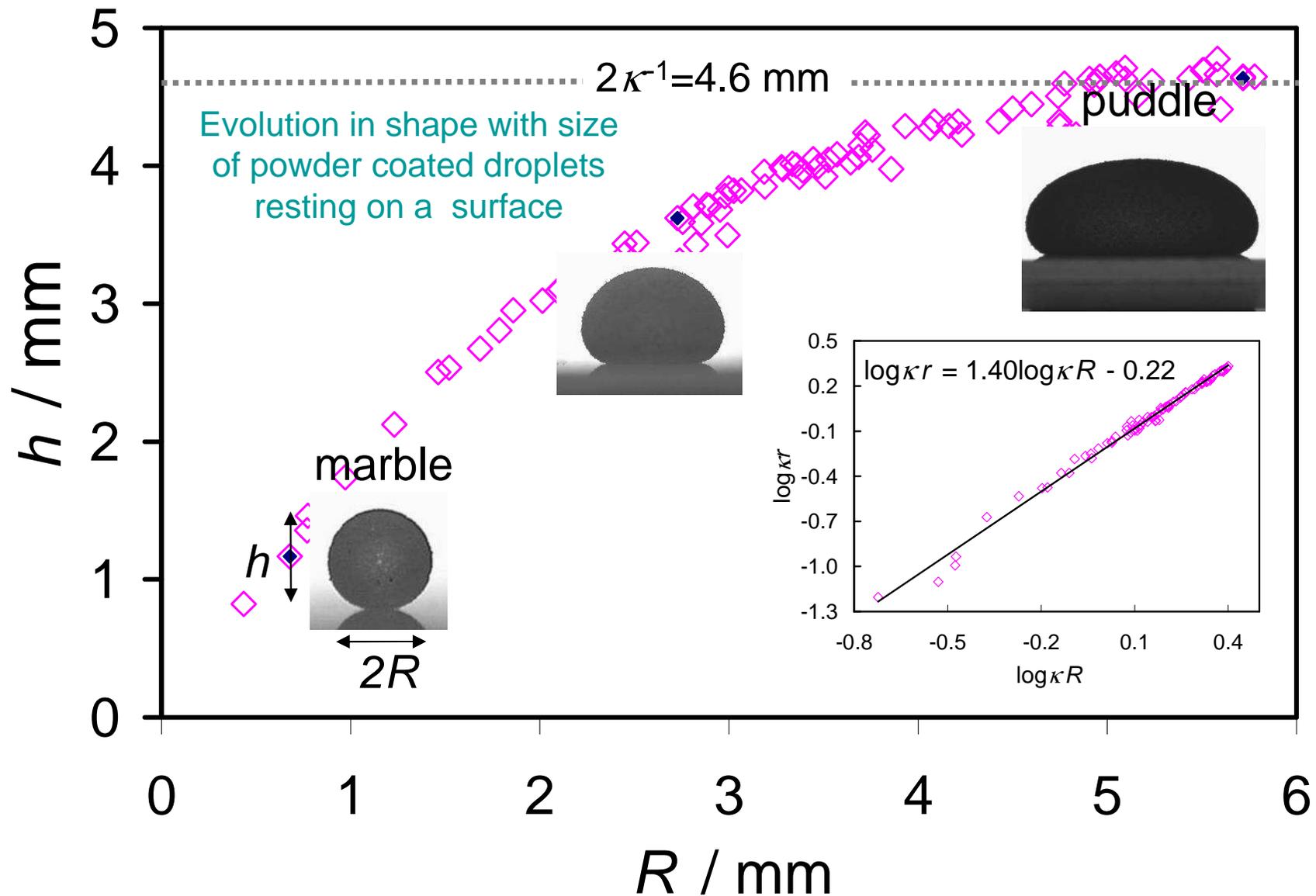
EU COST Action D19 - Chemistry at the nanoscale

EU COST Action P21 - Physics of droplets



Appendices

Liquid Marble Size Data (Lycopodium)



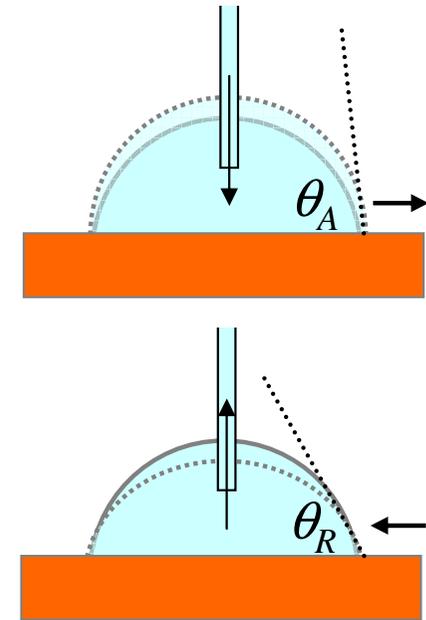
References Aussillous P, Quéré D. Proc. Roy. Soc. A462 (2006) 973-999; Nature 411 (2001) 924-927.

06 September 2009 McHale G., et al. 23 Langmuir (2007) 918-924. Newton, M.I., et al. J. Phys. D40 (2007) 20-24. 47

Contact Angle Hysteresis

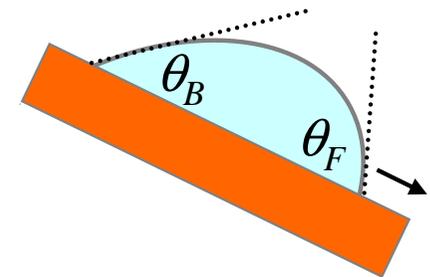
Advancing and Receding Contact Angles

- Largest θ prior to contact line motion as liquid fed in is θ_A
- Smallest θ prior to contact line motion as liquid withdrawn is θ_R
- Difference is contact angle hysteresis $\Delta\theta = \theta_A - \theta_R$
- In some sense characterizes difficulty of moving a droplet on a given “smooth and flat” surface



Tilt and Sliding Angles

- Tilt platform and measure forward, θ_F , and backward, θ_B , contact angles
- At instant before motion assume these give advancing and receding angles
- There is no proof that these are equivalent
- Sliding angle is lowered by superhydrophobicity¹

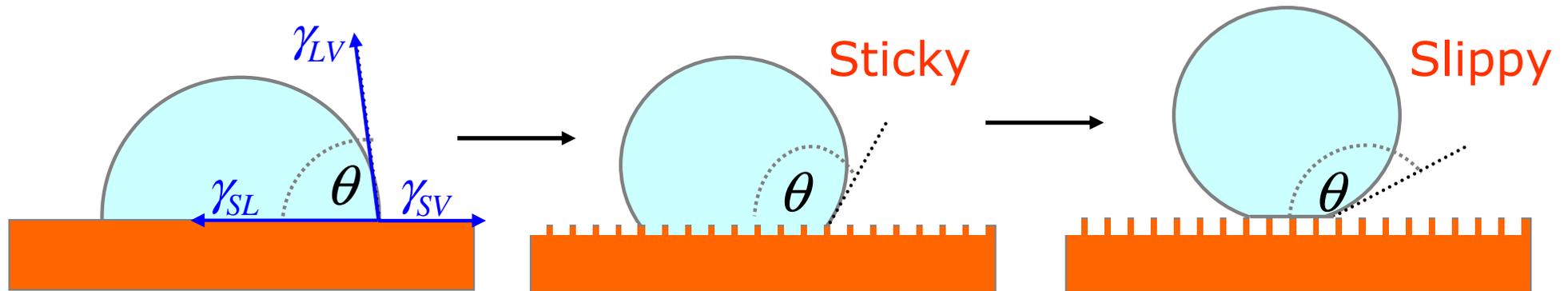


Reference ¹Miwa, M. *et al.*, Langmuir 16 (2000) 5754-5760.

Topography & Wetting

Droplets that Impale and those that Skate

What contact angle does a droplet adopt on a "rough" surface?



Young's Eq.

$$\cos \theta_e = (\gamma_{SV} - \gamma_{SL}) / \gamma_{LV}$$

Chemistry

Force view:

$$\gamma_{SL} + \gamma_{LV} \cos \theta_e = \gamma_{SV}$$

Wenzel Eq.

$$\cos \theta_W = r \cos \theta_e$$

Roughness

r = true area/planar projection

Cassie-Baxter Eq

$$\cos \theta_{CB} = f_s \cos \theta_e - (1 - f_s)$$

Topography

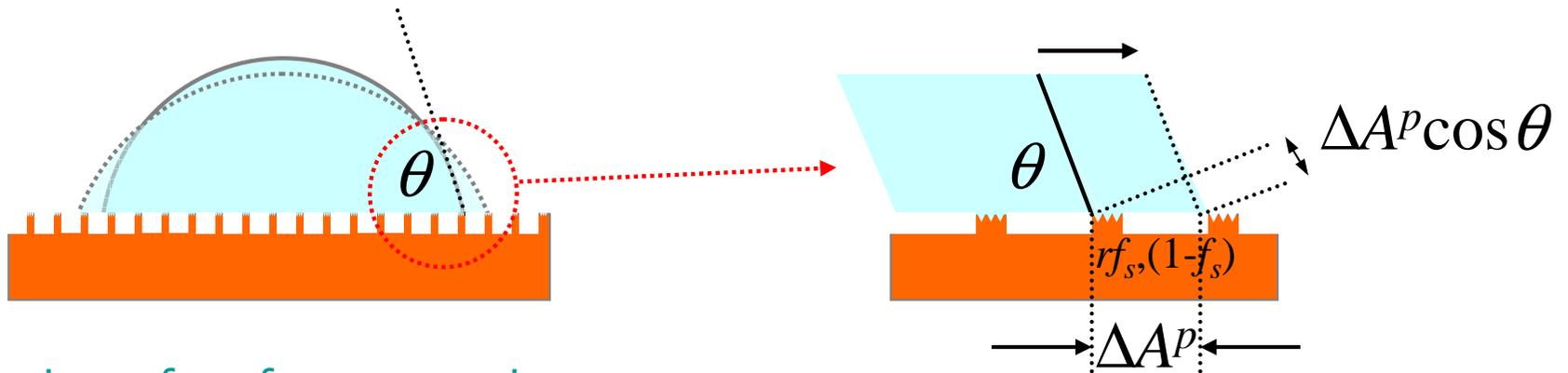
Young's Eq. θ_e

f_s = solid surface fraction

References Cassie, A. B. D.; Baxter, S. Trans. Faraday Soc. 40 (1944) 546-551. Wenzel, R. N.

Ind. Eng. Chem. 28 (1936) 988-994; J. Phys. Colloid Chem. 53 (1949) 1466-1467.

Topography 4: Top-Filled Dual Scale Surfaces



Change in surface free energy is

$$\Delta F = (\gamma_{SL} - \gamma_{SV}) r f_s \Delta A^P + \gamma_{LV} (1 - f_s) \Delta A^P + \gamma_{LV} \Delta A^P \cos \theta$$

Equilibrium is when $\Delta F = 0 \Rightarrow \cos \theta_{CB} = r f_s (\gamma_{SV} - \gamma_{SL}) / \gamma_{LV} - (1 - f_s)$

$$\cos \theta_{Obs} = f_s r \cos \theta_e - (1 - f_s)$$

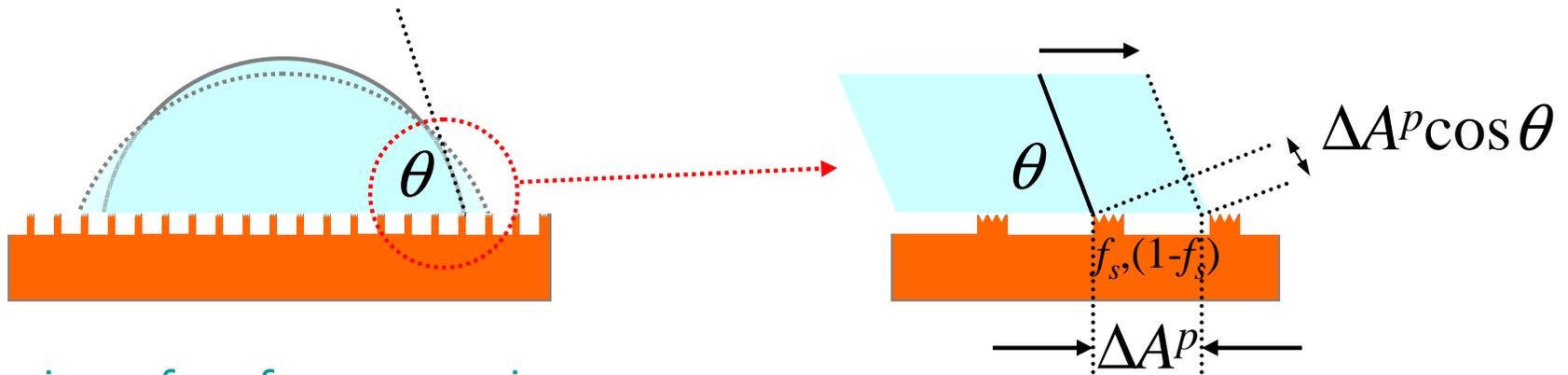
Topography $\Rightarrow f_s = \Delta A_{SL}^P / (\Delta A_{SL}^P + \Delta A_{LV}^P)$ = solid surface fraction from planar projections

$r = \Delta A_{SL} / \Delta A_{SL}^P$ = roughness of "tops" of features

Transformation via Wenzel eq. and then by Cassie-Baxter equation

$$\theta_e \rightarrow \theta_w(\theta_e) \rightarrow \theta_{CB}(\theta_w)$$

Topography 5: Top-Empty Dual Scale Surfaces



Change in surface free energy is

$$\Delta F = (\gamma_{SL} - \gamma_{SV}) f_s^{large} f_s^{small} \Delta A^P + \gamma_{LV} [(1 - f_s^{large}) \Delta A^P + f_s^{large} (1 - f_s^{small}) \Delta A^P] + \gamma_{LV} \Delta A^P \cos \theta$$

Equilibrium is when $\Delta F = 0 \Rightarrow$

$$\cos \theta_{Obs} = f_s^{large} [f_s^{small} \cos \theta_e - (1 - f_s^{small})] - (1 - f_s^{large})$$

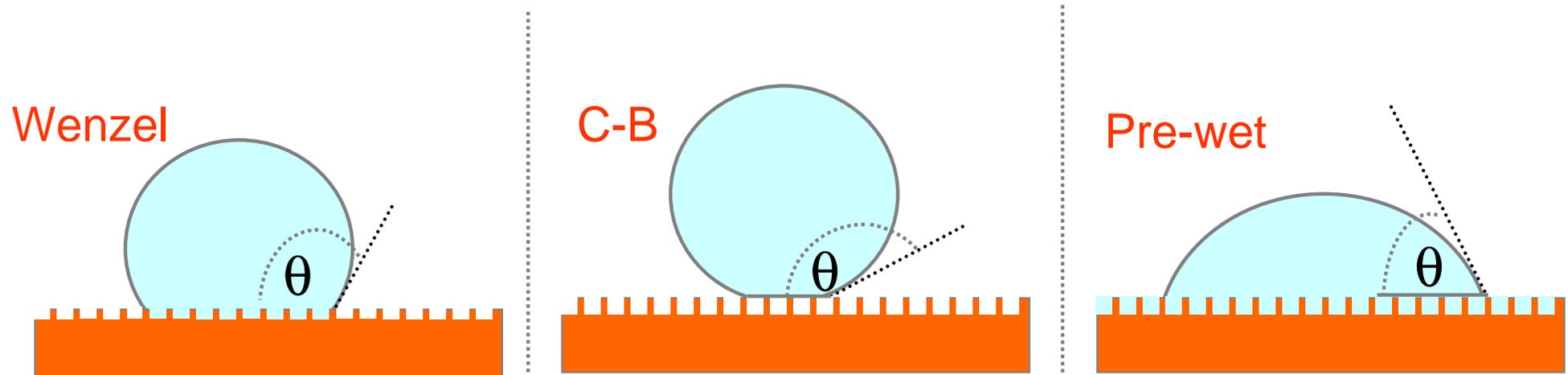
Topography $\Rightarrow f_s^{small}$ = solid surface fraction for small scale structure

f_s^{large} = solid surface fraction for large scale structure

Transformation via Cassie-Baxter and then by Cassie-Baxter again

$$\theta_e \rightarrow \theta_{CB}(\theta_e) \rightarrow \theta_{CB}(\theta_{CB})$$

Pre-existing Wetness



Weighted average of fractions f_s and $(1-f_s)$ with θ_e

ie. use $\cos(0^\circ)=+1$ in Cassie-Baxter equation

$$\cos \theta_{CB} = f_s \cos \theta_e + (1-f_s)$$

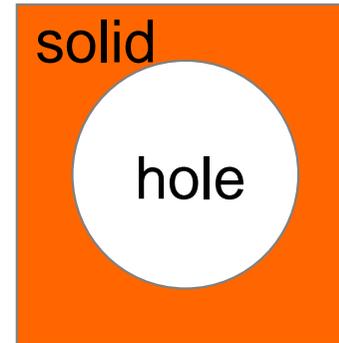
*sign has been switched to
+ve from -ve*

Cylindrical Model for Capillary Infiltration

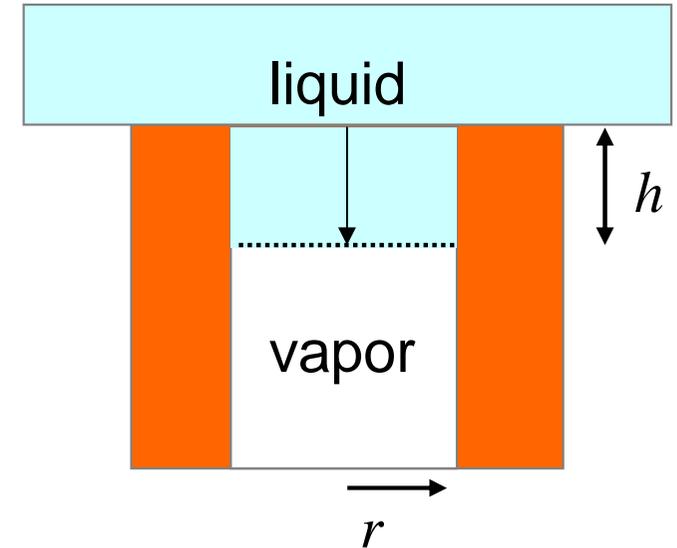
Assumptions

1. Fixed cylindrical pipe
2. Meniscus with Young's eq. contact angle, $\cos\theta_e = (\gamma_{SV} - \gamma_{SL})/\gamma_{LV}$
3. Minimise surface free energy, F

Top View



Side View



Change in surface free energy	=	solid-liquid energy per unit area	×	gain of wall area	minus	solid-vapor energy per unit area	×	loss of wall area
-------------------------------	---	-----------------------------------	---	-------------------	-------	----------------------------------	---	-------------------

$$\Delta F = (\gamma_{SL} - \gamma_{SV})2\pi r \Delta h$$

Young's eq.
 \Rightarrow
 $\Delta F = -\gamma_{LV} \cos\theta_e 2\pi r \Delta h$

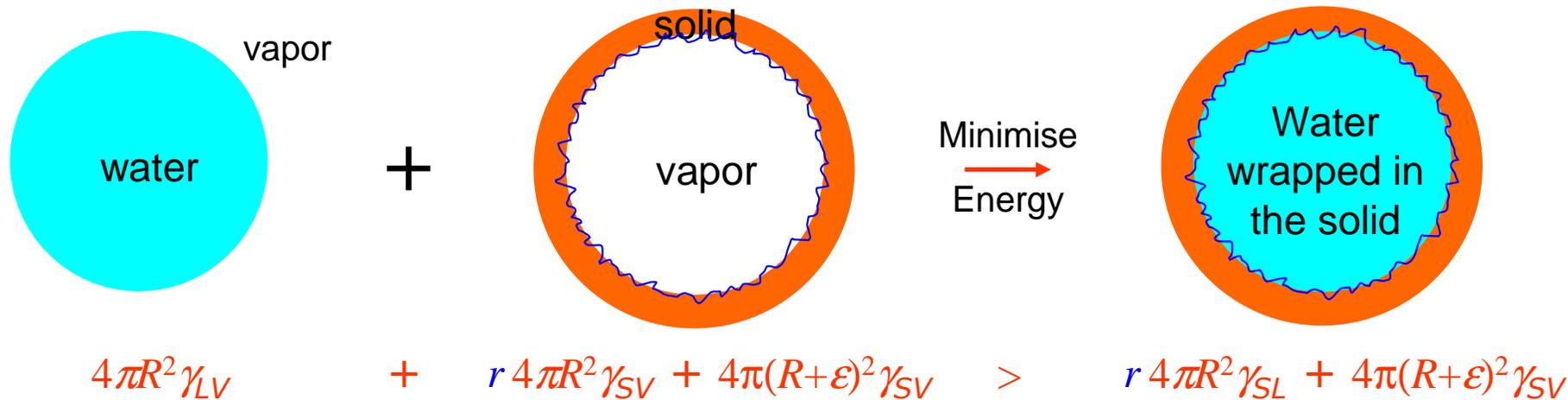
Spontaneous infiltration when ΔF is negative $\Rightarrow \theta_e < 90^\circ$

Same result for wetting down sides of posts on a superhydrophobic surface

Aren't all Solids with $\theta_e < 180^\circ$ Hydrophilic?

1. Assume energy in deforming solid is zero
2. Assume solid is smooth
3. Under these conditions surface free energy always favors solid wrapping up a droplet providing the Young's eq. contact angle is less than 180°

Hydrophobic Solid Shell (of thickness ε) and Water



gives $\Delta F/4\pi R^2 = r \gamma_{SL} - \gamma_{LV} - r \gamma_{SV}$ Use Young's eq. $\Rightarrow = -(1 + r \cos \theta_e) < 0 \Rightarrow \theta_e < \theta_e < 90^\circ \quad r \rightarrow \infty$

All smooth ($r=1$) solids with Young's eq. $\theta_e < 180^\circ$, incl. Teflon, are absolutely hydrophilic, although those with $\theta_e > 90^\circ$ have a tendency to hydrophobicity (in a Wenzel sense)

Bead Pack/Soil Model Calculations

Surface Free Energy Considerations

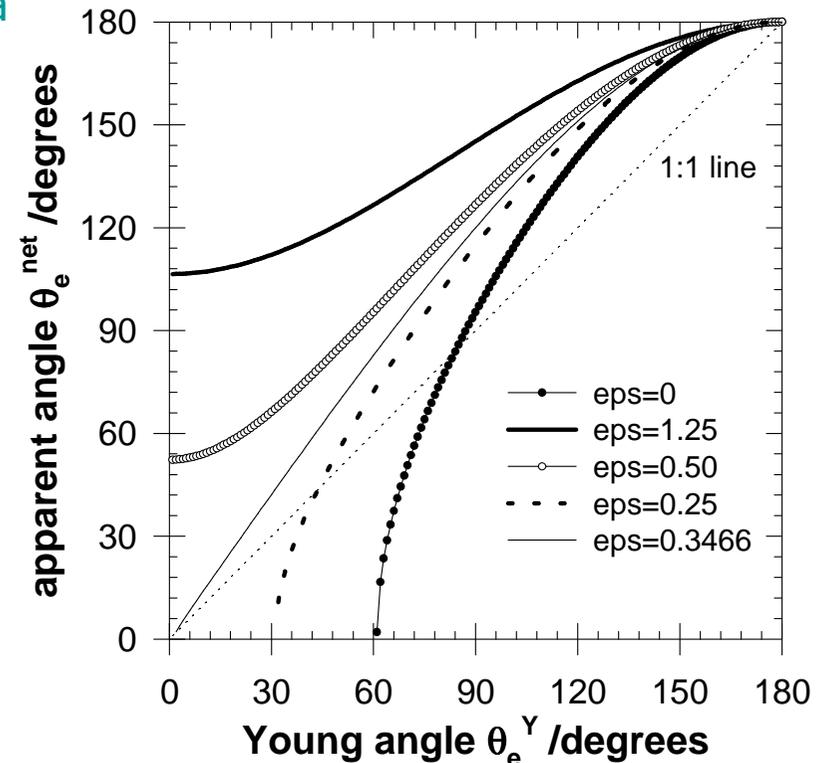
1. the curved bead surface effectively gives a roughness factor, r_s
2. the planar projection of the bead and the gap between beads forms a Cassie-Baxter system with a solid surface fraction, f_s
3. both r_s and f_s depend on the chemistry (via Young's eq.)
4. Young's contact angle is converted to a Wenzel contact angle and then to a Cassie-Baxter contact angle

Equations

$$\theta_e \xrightarrow{\text{Wenzel}} \theta_W \xrightarrow{\text{Cassie-Baxter}} \theta_{CB}$$

$$\cos \theta_e^{net} = f_s r_s \cos \theta_e - (1 - f_s)$$

$$f_s = \frac{\pi \sin^2 \theta_e}{2\sqrt{3}(1 + \epsilon)^2} \quad r_s = \frac{2(1 + \cos \theta_e)}{\sin^2 \theta_e}$$



References: McHale, G. *et al.*, *Eur. J. Soil. Sci.* **56** (2005) 445-452. Bachmann, J. & McHale, G.

06 September 2009 *Eur. J. Soil Sci.* **60** (2009) 420-430.